H. E Brown

ELECTRICAL ENGINEERING

N72-12882 (NASA-CR-121031) TIME DOMAIN AND FREQUENCY DOMAIN DESIGN TECHNIQUES FOR MODEL REFERENCE ADAPTIVE CONTROL SYSTEMS J.S. Boland, III (Auburn Univ.) 15 Oct. 1971 173 p CSCL 22B G3/31

Unclas
09408

ENGINEERING EXPERIMENT STATION

AUBURN UNIVERSITY

AUBURN, ALABAMA

TIME-DOMAIN AND FREQUENCY-DOMAIN DESIGN TECHNIQUES FOR MODEL-REFERENCE ADAPTIVE CONTROL SYSTEMS

PREPARED BY

GUIDANCE AND CONTROL STUDY GROUP

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SECOND TECHNICAL REPORT

OCTOBER 15, 1971

CONTRACT NAS8-26580
GEORGE C. MARSHALL SPACE FLIGHT CENTER
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
HUNTSVILLE, ALABAMA

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FOREWORD

This report is a technical summary of the progress made by the Electrical Engineering Department, Auburn University, toward fulfillment of Contract NAS8-26580 granted to Auburn Research Foundation, Auburn, Alabama. This contract was awarded November 15, 1970, by the George C. Marshall Space Flight Center, National Aeronautics and Space Administration, Huntsville, Alabama.

ACKNOWLEDGEMENT

A dissertation to be submitted by Donald W. Sutherlin to the Graduate Faculty of Auburn University in partial fulfillment of the requirements for the degree of Doctor of Philosophy is based on the work reported herein.

The research for this dissertation was performed under Contract NAS8-26580 awarded to the Auburn Engineering Experiment Station, Auburn, Alabama, by the George C. Marshall Space Flight Center of the National Aeronautics and Space Administration. The project leader for this work was Assistant Professor Joseph S. Boland, III, of the Electrical Engineering Department, Auburn University.

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SUMMARY

Some problems associated with the design of model-reference adaptive control systems are considered and solutions to these problems are advanced. The stability of the adapted system is a primary consideration in the development of both the time-domain and the frequency-domain design techniques. Consequentially, the use of Liapunov's direct method forms an integral part of the derivation of the design procedures. The application of sensitivity coefficients to the design of model-reference adaptive control systems is considered. An application of the design techniques developed herein is also presented.

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I. INTRODUCTION

The control requirements necessary for many of today's complex physical processes has fostered the search for control methods which provide better performance characteristics than could be obtained with conventional control methods. For example, the use of computers in the control loop of complex industrial processes is commonplace. In addition, considerable research effort is being expended in order to provide systems with the capabilities to "adapt" themselves to a changing environment so as to maintain satisfactory performance characteristics throughout the entire environmental profile. The need for satisfactory performance of a control system in a changing environment or with inadequately defined parameters is not new. Indeed, one of the reasons for the use of feedback control systems is the inherent capability of the system to be somewhat insensitive to changes in the controlled process. However, for some physical processes, conventional methods are inadequate and other methods of control are necessary.

The interest in adaptive control has been generated largely because of a number of problems which could not be solved using conventional design techniques. Such a situation might develop when the plant parameters change grossly during the operation of the system. To further complicate the problem, the environment in which the plant is operating may also be changing drastically. These variations will not, in general, be deterministic. Some may be predicted by statistical methods while others may be totally unpredictable. Specifically, high-performance aircraft and missiles have widely-varying parameters and in many cases operate in extremely different environmental characteristics during the operation profile of the mission. Some extensive bibliographies of research in the adaptive control area have been compiled [1], [2], [3]. In addition to a general adaptive control bibliography, reference [3] also contains compilations of research effort in particular areas of adaptive control such as adaptive process controllers and identification.

There appears to be no universally accepted definition for an adaptive control system. Henceforth, an adaptive control system will be defined as:

Definition I-1. An adaptive control system is a system which monitors some of the plant characteristics, compares this with the desired characteristics and uses the difference to provide adaptation so that the desired performance may be obtained.

There are, essentially, two basic approaches to the adaptive control problem. One approach which may be referred to as an "open-loop" policy is the preprogrammed adaption scheme. In this approach the plant description, the environmental conditions, and a performance criterion are assumed to be known. The adaptive parameters are then determined so as to be optimum with respect to the given performance index. Environmental measurements are then made and parameters adjusted based on these measurements. For proper operation, this scheme requires accurate knowledge of the plant and extensive information regarding the relationships of the measured environmental quantities to the system characteristics. These relationships as well as accurate identification of the plant may well be very difficult to ascertain.

The second approach, which is of primary interest, may be referred to as "closed-loop". Here, the performance criterion is continuously monitored and this information is used to adapt the system so that the desired performance characteristics are obtained. This "closed-loop" property provides increased system reliability since the adaptive scheme has the capability of achieving satisfactory performance despite failure of some of the system components. The "closed-loop" concept of adaptive control systems is of major concern and will be considered in the following.

A. Classification of Adaptive Control Schemes

Adaptive control systems may take many different forms. Three classifications of adaptive control systems which encompass most of the different schemes are

(1) High-gain schemes,

(2) Optimal adaptive methods, and

(3) Model-reference adaptive control systems.

In general, the high-gain schemes are the simplest types of adaptive control systems while the optimal adaptive methods are the most complex of the three classifications.

1. High-Gain Adaptive Control Systems

The basic premise behind the high-gain schemes is that if the loop gain around a plant is kept sufficiently high, the input-output transfer function is virtually independent of the plant dynamics.

This scheme is exemplified in Figure I-1. From Figure I-1, the transfer function from S to C is

$$\frac{C(s)}{S(s)} = \frac{KG_p(s)}{1 + KG_p(s)} \tag{I-1}$$

Assuming that

$$|KG_p(s)| >> 1, \tag{I-2}$$

the transfer function from S to C is

$$\frac{C(s)}{S(s)} \simeq 1 \tag{I-3}$$

Under this assumption, the transfer function from the actual input R to the plant output C is

$$\frac{C(s)}{R(s)} \simeq G_m(s) \tag{I-4}$$

Thus, the transfer function from input to output is approximately equal to the arbitrarily specified model transfer function and is essentially independent of the plant dynamics. In practical systems, however,

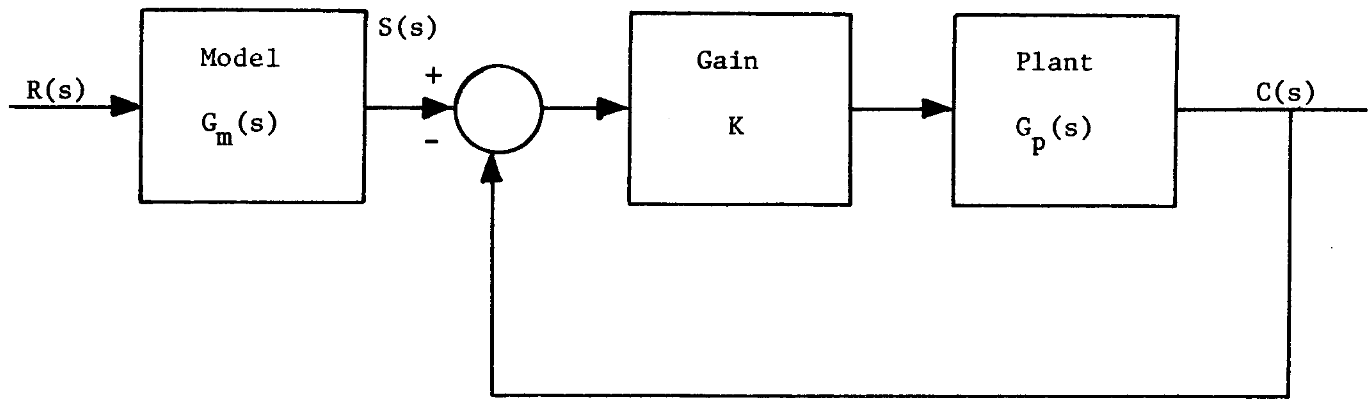

Figure I-1. Block diagram of basic high-gain adaptive control scheme.

stability problems arise with increasing values of gain. Because of this, it is necessary to monitor the signal in the plant loop and insure that the system remains stable. With this information, the gain, K, may be adjusted so the system is on the verge of instability. With the gain as high as practically possible because of stability reasons, the transfer function approximation of equation (I-4) is as accurate as possible. The block diagram of a practical high-gain adaptive control system is shown in Figure I-2. Here, the loop stability monitoring characteristics and the gain adjusting capabilities are included.

The high-gain concept of adaptive control does have serious drawbacks, however. One of the major disadvantages of the high-gain approach is that a considerable amount of apriori information about the system must be known. For example, from a practical standpoint, in order to utilize the capabilities of the approach, the approximate value of gain where the roots of the system characteristic equation go into the right-half plane must be known.

One of the first applications of the high-gain approach for adaptive control systems was the adaptive system for the horizontal stabilizers on the X-15 aircraft [4]. The system block diagram, not including the reference model, for the roll axis is shown in Figure I-3. As can be seen from the figure, the gain in the system is a function of a fixed gain, K_1, and a variable stability augmentation system (SAS) or adaptive gain, K.

The adaptive gain, K, was varied so as to provide optimum system performance throughout the various regions of dynamic pressure in which the X-15 would be operating. Adaptive operation was maintained at an

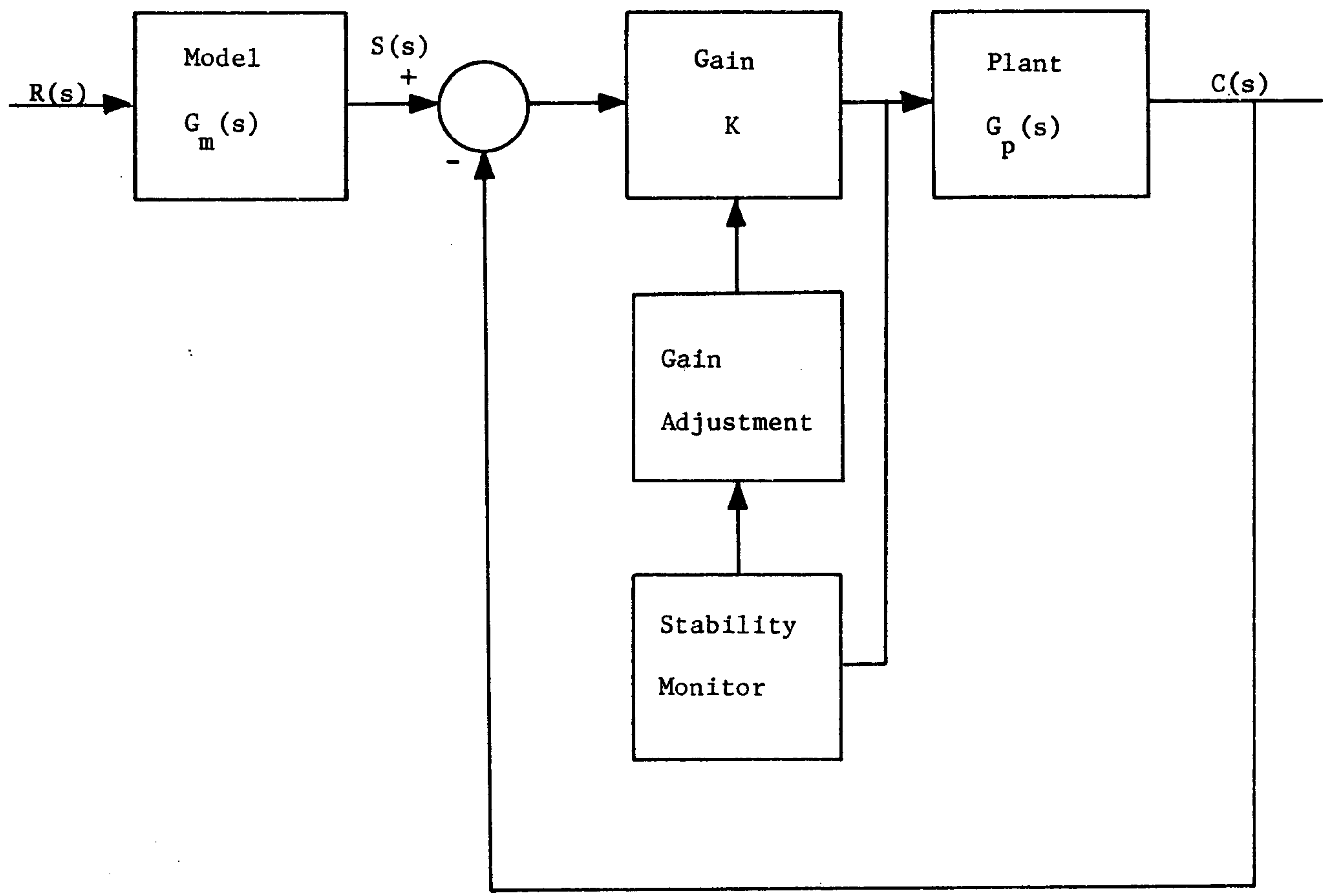

Figure I-2. Block diagram of practical high-gain adaptive control scheme.

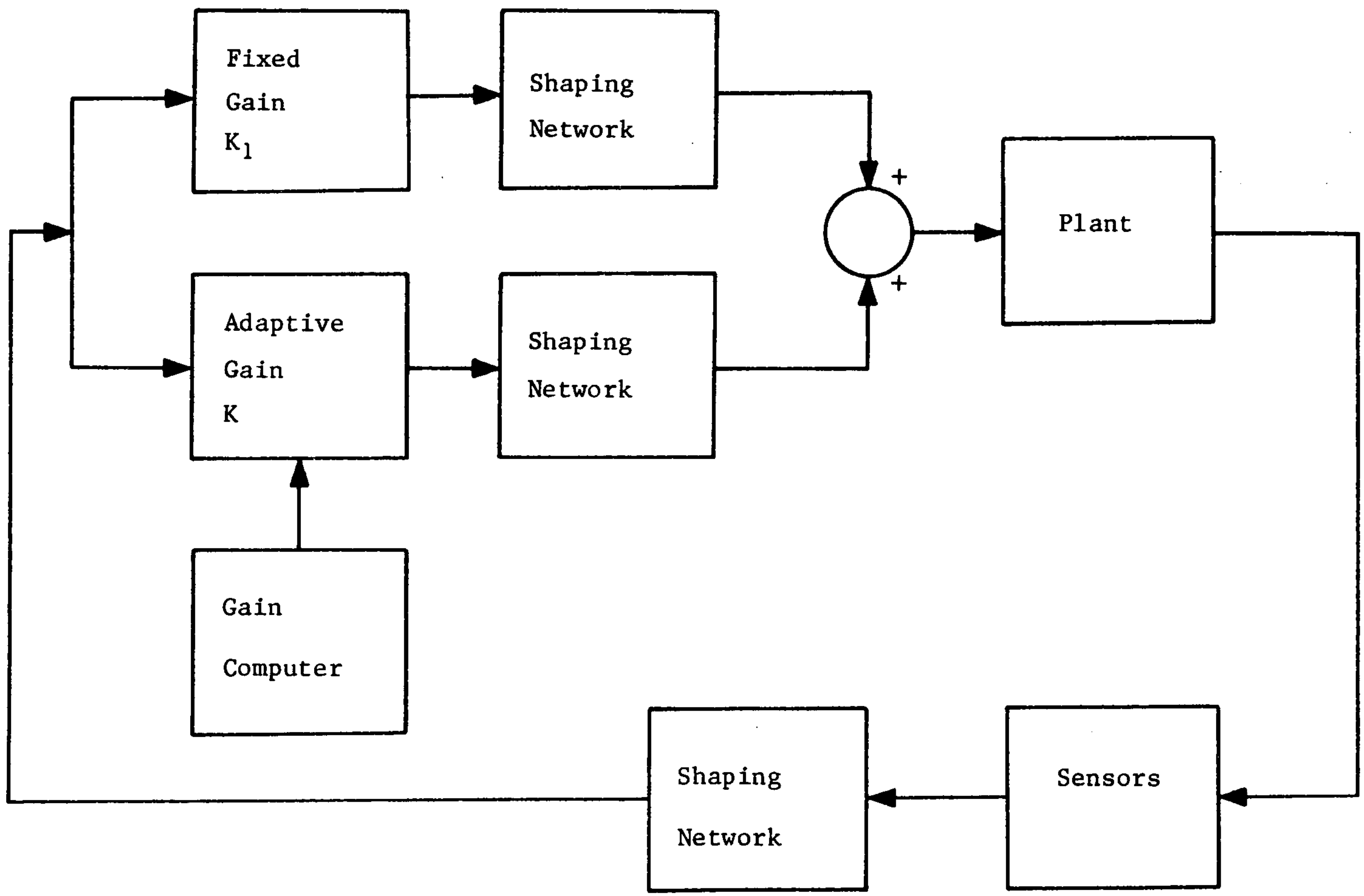

Figure I-3. Roll axis block diagram for X-15 aircraft.

oscillation frequency of 4 hz. The various shaping networks were designed to compensate for effects such as structural bending modes and a resonant mode due to the mass unbalance of the stabilizers. This high-gain adaptive control system was successfully flight-tested with the X-15 aircraft. With the adaptive system, the response to a pulse or step input in the roll and yaw axis was essentially critically damped whereas the pitch axis response to a pulse or step input had a slight overshoot.

2. Optimal Adaptive Control Systems

The optimal adaptive control method is, in general, the most complex of the adaptive control schemes. In the optimal adaptive approach, the adaptive action is determined in order to achieve a maximum or a minimum of some prespecified performance index whereas in other approaches such as the high-gain scheme, the adaptive action consists of attempting to match some prespecified system characteristics such as the system pole-zero locations. Thus, the optimal adaptive control method basically consists of solving some optimization problem. However, in order to solve the optimization problem, it is necessary to assume that the system parameters and system states are known. In general, for an adaptive control problem, these quantities are not completely known and plant identification and state estimation become an integral part of the problem. However, this adds measurably to the complexity of the adaptive control problem. A general block diagram for an optimal adaptive control system is shown in Figure I-4.

For plant identification, correlation techniques may be used in order to obtain an approximation to the system inpulse response. The

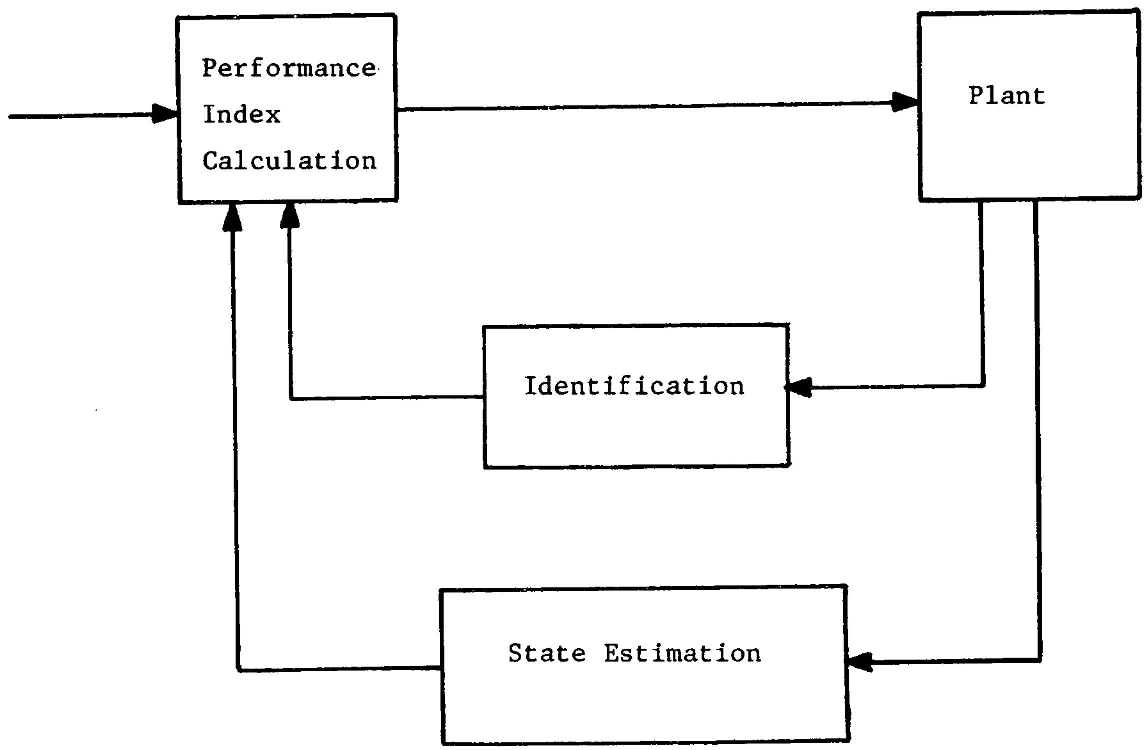

Figure I-4. Block diagram for basic optimal adaptive control scheme.

input-output cross-correlation function, $R_{xy}(\tau)$, for a system with an impulse response, h(t), which is excited by a noise signal with autocorrelation, $R_{xx}(\tau)$, is given by the expression:

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} R_{xx}(\tau + t)\ h(t)\ dt \tag{I-5}$$

If the input is white noise or if the bandwidth of the input is considerably larger than the system bandwidth, the autocorrelation function may be represented as an impulse and the input-output cross-correlation is given by

$$R_{xy}(\tau) \simeq g(-\tau). \tag{I-6}$$

Under the assumption of ergodicity, the cross-correlation function may be approximated by a time average. One point of the impulse function may then be obtained for each value of delay, τ, at which the input-output cross-correlation is performed. By operating a number of the input-output cross-correlators in parallel with delays of τ, 2τ, 3τ, etc., a reasonable approximation to the system impulse response may be obtained.

One technique for the solution of the state-estimation problem is with the methods discussed in Meditch[5]. Consider the system represented by the relations

$$\dot{\underline{x}}(t) = F(t)\ \underline{x}(t) + G(t)\ \underline{w}(t) \tag{I-7}$$

$$\underline{z}(t) = H(t)\ \underline{x}(t) + \underline{v}(t), \tag{I-8}$$

where $\underline{x}(t)$ is the system state, an n-vector, $\underline{z}(t)$ is the measurement vector, an m-vector, and $\underline{w}(t)$ and $\underline{v}(t)$ are zero mean gaussian white noise vectors of dimension p and m, with covariance matrices Q(t) and R(t), respectively. F(t), G(t), and H(t) are matrices of dimension nxn, nxp, and mxn, respectively. A block diagram of this system is shown in Figure I-5. The problem of estimation may then be stated as follows:

Given the measurement $\underline{z}(t)$, we wish to find an estimate of $\underline{x}(t)$, which will be denoted as $\hat{\underline{x}}(t)$. This estimate will be defined as an n-dimensional function, ϕ, of the measurements. In equation form, this may be stated as:

$$\hat{\underline{x}} = \phi[\underline{z}(t)] \tag{I-9}$$

The filtered estimate for the system of Equation (II-7) and (II-8), first solved by Kalman, is given by

$$\dot{\hat{\underline{x}}} = F(t)\ \hat{\underline{x}} + K(t)[\underline{z}(t) - H(t)\ \underline{x}], \tag{I-10}$$

where K(t) is an nxm filter gain matrix given by

$$K(t) = PH^T(t)\ R^{-1}(t), \tag{I-11}$$

where $(\ )^T$ and $(\ )^{-1}$ denote transpose and inverse, respectively, and P is the covariance matrix of the filter estimation error and is the solution of the matrix Riccati equation

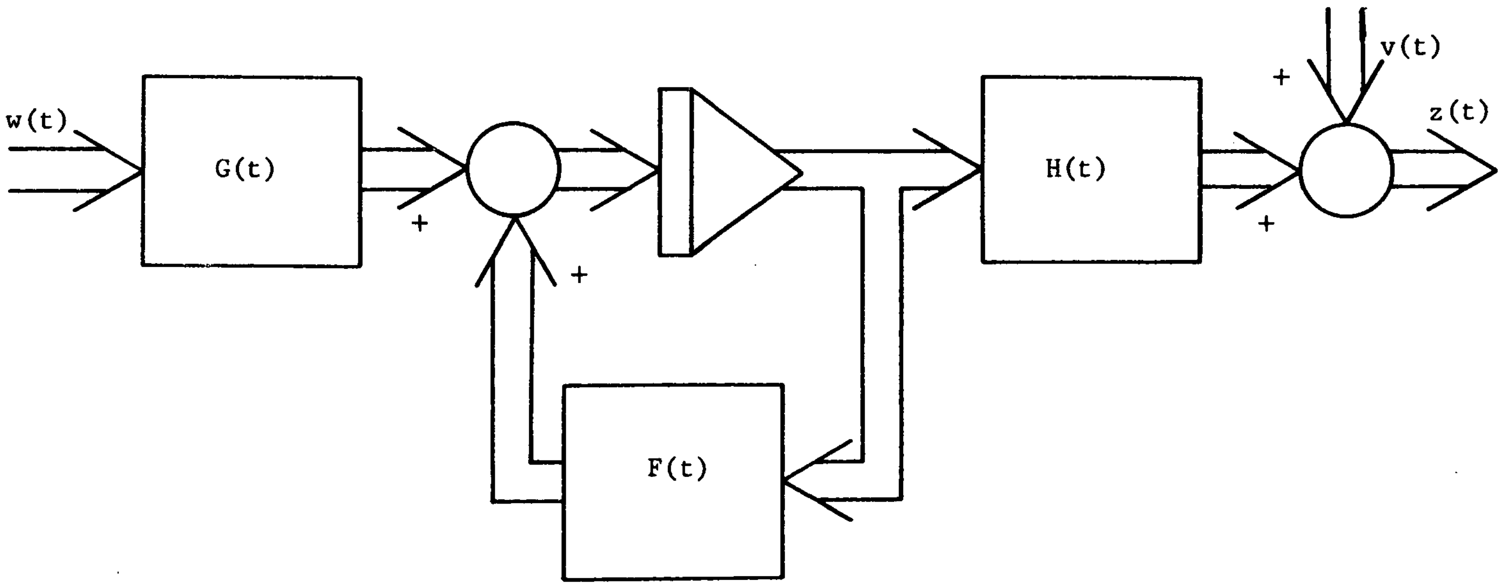

Figure I-5. Block diagram of system for state estimation.

$$\dot{P} = F(t)\,P + PF^T(t) - PH^T(t)\,R^{-1}(t)\,H(t)\,P + G(t)\,Q(t)\,G^T(t) \qquad \text{(I-12)}$$

The preceding methods for plant identification and for state estimation require a large amount of calculations and for large order systems, the computational requirements may be excessive. The delays introduced by the required computations may, in many instances, cause stability problems. In addition, the plant identification scheme requires an auxiliary input to the system which consists of noise with a bandwidth much greater than the bandwidth of the system. This auxiliary input may, in many applications, be impractical.

One example of an optimal adaptive scheme is that of Margolis [6]. In this scheme, identification is accomplished by having a "learning" model which is an analog of the system to be adapted. The model contains adjustable parameters which are used to force the model to dynamically track the system. The same input signals are received by both the model and the plant and the difference in the plant and model outputs is used to determine the necessary parameter adjustments. The adjusted parameters of the model are then used to compute the plant alterations necessary in order to provide minimization of the specified Performance Index.

3. Model-Reference Adaptive Control Systems

The concept of model-reference adaptive control evolved at the Massachusetts Institute of Technology Instrumentation Laboratory [7]. It evolved from a research program to develop automatic flight control systems capable of providing adequate control throughout a very wide operating range. The adaptive action is "closed-loop" and adaptation is performed based on normal operating inputs to the system. Hence, special test signals for disturbing the system are not required. A basic block diagram of a model-reference system is shown in Figure I-6.

In the model-reference concept, system performance is evaluated by comparing it with the performance of a reference model. The reference model is designed so that its output, when excited by the same input commands as to the system to be adapted, gives the desired response of the system. The response error is formed by taking the difference between the output of the reference model and the output of the plant. The adaptive mechanism then determines the adaption necessary so that the system response closely approximates that of the model.

Two of the approaches proposed in order to achieve the required adaptation for model-reference adaptive control systems are those of Osburn [8] and Dressler [9]. In the work by Osburn, the basic configuration is as shown in Figure I-6. The plant is assumed to have k adaptive parameters. The performance criterion, L, is an even function of the k adaptive parameters. L is then used in order to determine how the adaptive parameters should be adjusted in order to achieve the desired system performance. The adjustments of the adaptive parameters are determined in the following manner. The performance criterion, L, which is a function of the k adaptive parameters is viewed as a hyperplane in the k-dimensional adaptive parameter space. The objective then is to find the required adaptive parameter values so that the performance criterion is

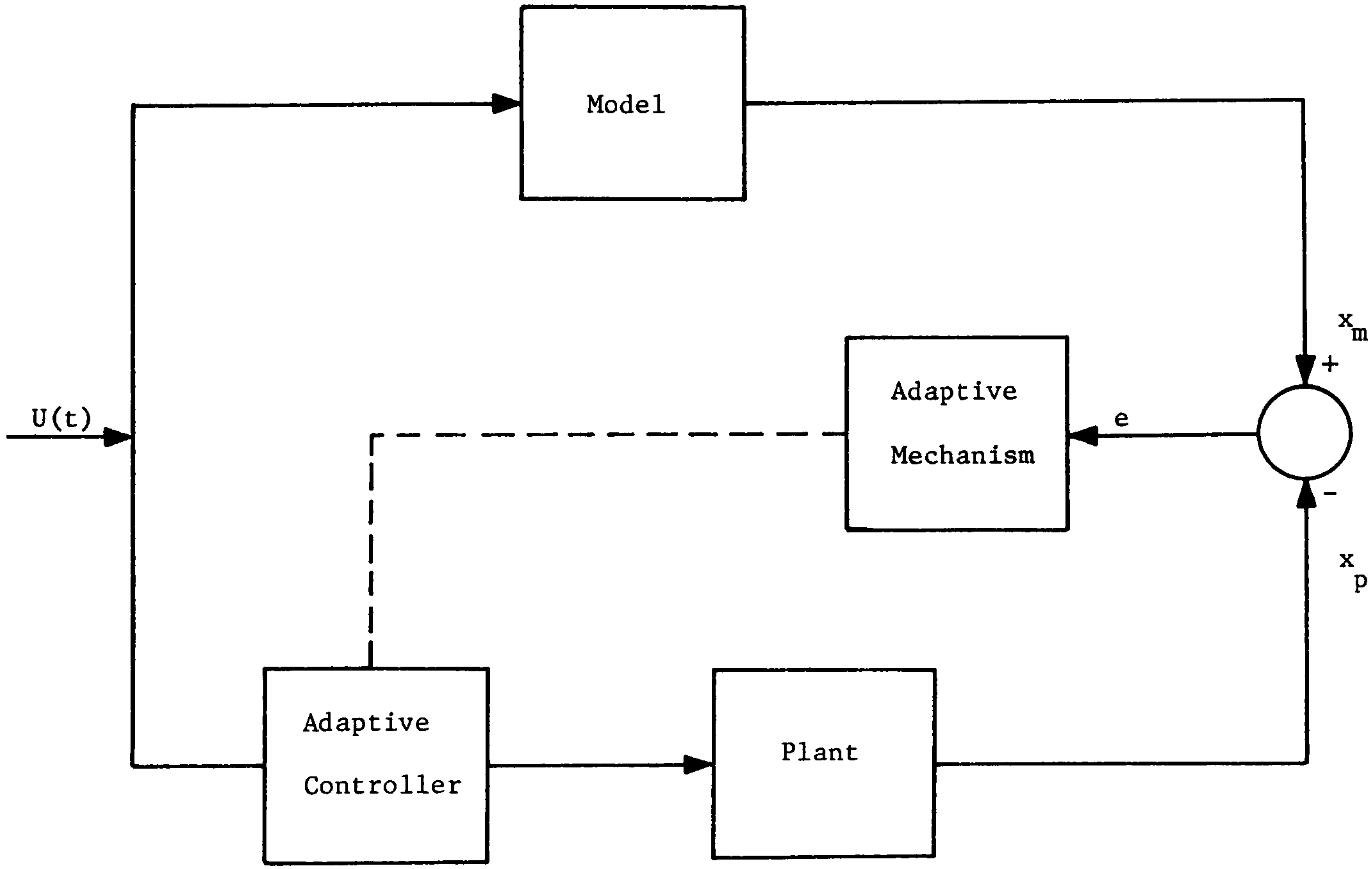

Figure I-6. Block diagram of basic model-reference adaptive control scheme.

minimized. The partial derivatives of the performance criterion with respect to the various adaptive parameters are obtained. Each parameter is then adjusted at a rate proportional to the partial derivative of L with respect to that parameter. The adaption then proceeds to an extremum along the path of steepest descent. The particular performance criterion selected by Osburn is

$$L = \int_{t_1}^{t_2} e^2 \, dt. \tag{I-13}$$

The interval $[t_1, t_2]$ must be of sufficient length so as to include a large portion of the system response due to an input at time t_1. The parameter adjustment of P_k, the k^{th} adaptive parameter, is given by:

$$\Delta P_k = - K \frac{\partial}{\partial P_k} \int_{t_1}^{t_2} e^2 \, dt \tag{I-14}$$

where K is an adaptive gain constant which determines the magnitude of the adjustment.

The selection of a performance criterion for use with the method of steepest ascent or descent is very critical. For example, if the hyperplane represented by the performance criterion has several points at which the gradient is zero, but at some of these points, the performance criterion does not assume its absolute minimum or maximum value, then the adaptive parameters may not be adjusted to the values necessary to attain the desired system characteristics. Also, the complexity associated with the generation of the necessary partial derivatives is a distinct disadvantage of this method.

Dressler approaches the model-reference adaptive control problem from the state-space point of view. The model system is assumed to be of the form

$$\dot{\underline{x}}_m = A_m \underline{x}_m + B_m \underline{u} \qquad \text{(I-15)}$$

where $\underline{x}_m$ is an n^{th} order model state vector, $\underline{u}$ is an r-dimensional input vector, and A_m and B_m are matrices of order nxn and nxr, respectively. The plant is assumed to be represented by:

$$\dot{\underline{x}}_p = A_p \underline{x}_p + B_p \underline{u} \qquad \text{(I-16)}$$

where $\underline{x}_p$ is an n^{th} order plant state vector, $\underline{u}$ is the same input as in the model, and A_p and B_p are nxn and nxr matrices, respectively. In order to obtain analytically the explicit functional relationship between the response error and the adaptive parameters it is assumed that the matrices A_p and B_p may be decomposed as follows:

$$A_p = A_m + \delta A_\delta \qquad \text{(I-17)}$$

$$B_p = B_m + \delta B_\delta$$

The matrices δA_δ and δB_δ contain the adaptive parameters and are considered as perturbations of the plant matrices from the model matrices.

With these assumptions, an explicit relationship for the response error may be derived using the method of successive approximations for the solution of the necessary differential equations. In order to obtain a measure of the change in the response error due to a change in the adaptive parameters, the incremental response error is defined as

$$\Delta e(t) \triangleq e(t + \Delta t) - e(t) \tag{I-19}$$

where Δt is chosen sufficiently small so that any incremental response error occurring in Δt time is due only to the change caused by the adjustment of the adaptive parameters.

The basic premise in Dressler's solution to the adaptive control problem may be stated thusly: If at some time t_1, the response error is not zero, the adaptive parameters will be adjusted so as to cause the magnitude of the response error to be decreased for $t > t_1$. In equation form, the design criterion may be stated as:

If	then
$e(t_1) < 0$	$\Delta e(t_1 + \Delta t) > 0$
$e(t_1) > 0$	$\Delta e(t_1 + \Delta t) < 0$ and
$e(t_1) = 0$	$\Delta e(t_1 + \Delta t) = 0.$ (I-20)

Dressler shows that this design criterion may be satisfied if the adaptive parameter rates are chosen by the relationships:

$$\dot{a}(t) = k_1 \, x_m(t) \, e(t) \tag{I-21}$$

$$\dot{b}(t) = k_2 \, u(t) \, e(t) \tag{I-22}$$

where a and b are elements of A_p and B_p, respectively and k_1 and k_2 are adaptive gain constants.

It is obvious from comparing equations (II-14), (II-21), and (II-22) that the adaptation relationships obtained by Dressler would be much simpler to implement than those obtained by Osburn. However, in both of these techniques, the stability of the adaptive system is not considered in the design relationships.

B. Purpose of Present Research

It is the purpose of the present research to obtain adaption relationships for model-reference adaptive control systems which, by utilization of the stability theorems of Liapunov, incorporate stability considerations of the adaptive system into the basic design equations. Adaption relationships are derived from both time domain and frequency domain considerations. An investigation is also made into the use of sensitiity coefficients in connection with the design of model-reference adaptive control systems.

Specifically, Chapter II contains a detailed discussion of the determination of stability via Liapunov's direct method. In Chapter III, the adaption relationships are derived for the design of model-reference control systems using Liapunov's direct method. The design procedure is developed for a time-domain representation of the system. In

Chapter IV, the second method of Liapunov is used to design feedback filters and pre-filters to provide the necessary adaption. This procedure is based on frequency-domain considerations. In Chapter V, the use of sensitivity coefficients in the design of model-reference adaptive control systems is studied. Chapter VI contains a comparison of the various adaption schemes developed and conclusions regarding their use.

II. LIAPUNOV'S DIRECT METHOD

Since the use of Liapunov's direct method is vital to the design techniques developed in the present research, it is felt that a chapter elucidating the second method is necessary for completeness. The material contained in this chapter essentially comes from the author's class notes and [10], [11], and [12].

A. Historical Background

The direct method of Liapunov was introduced by the Russian mathematician A. M. Liapunov in a dissertation in 1892. The method is a general procedure for determining the stability of ordinary differential equations. It is of very broad scope in that it is applicable to autonomous or nonautonomous and to linear or nonlinear ordinary differential equations. In the literature, the direct method is sometimes referred to as Liapunov's second method. Liapunov classified the stability analysis techniques for ordinary differential equations as belonging to one of two methods. The indirect, or first method, requires the solution to the differential equations under investigation. The direct, or second method, which is of primary interest, does not require the solution of the differential equations under investigation, but deals with stability criteria which can be applied directly to these equations.

B. Liapunov Stability

Before discussing the determination of stability via Liapunov's direct method, it is necessary to properly and precisely define the concept of stability. In a linear, autonomous system, there will exist only one equilibrium state. If all system trajectories approach this equilibrium state as time approaches infinity the system is said to be stable. If all system trajectories not initially at the equilibrium state approach infinity as time approaches infinity, then the system is said to be unstable. An exception to these definitions is the case where the system has roots which are purely imaginary. For this case, if the system is not initially at the equilibrium state, the system will oscillate for all time. The stability of a linear system is independent of the magnitude of the initial conditions and, in addition, the output of the system is bounded for any bounded input. Unfortunately, for nonlinear systems, there are various degrees or levels of stability. There are at least 3 basic differences in the concept of stability for a nonlinear system. These are: (1) A nonlinear system may have several equilibrium states. Therefore, in general, it is necessary to speak of the stability relative to an equilibrium state rather than stability of the system as was done in the linear case; (2) In the nonlinear system stability of an equilibrium state does not imply stability of the overall state space. The notions of local stability and global stability must be considered separately; (3) Even in the case of local stability, the concepts of boundedness of the system state trajectories and the asymptoticity of the system state trajectories are involved. Boundedness of a system state trajectory refers to the property that a system state which is initially in some suitably restricted region, will never leave another well-defined region. The concept of asymptoticity refers to the property that all system state trajectories will approach the equilibrium state under investigation as time approaches infinity.

The basic idea of stability as formulated by Liapunov is of the same notion as the boundedness property discussed above. In the following, $|| \quad ||$ denotes the Euclidean norm. The definition for Liapunov stability is as follows:

Definition II-1. An equilibrium state, x_e, of an autonomous dynamic system is stable in the sense of Liapunov if for $\varepsilon > 0$, there exists a $\delta > 0$ such that if $||x_0 - x_e|| < \delta$, then $||x(t) - x_e|| < \varepsilon$ for all $t > t_0$.

The concept of Liapunov stability is illustrated in Figure II-1. The definition for not being Liapunov stable may be stated thusly:

Definition II-2. An equilibrium state, x_e, is not stable in the sense of Liapunov if there exists an $\varepsilon > 0$ such that if for all $\delta > 0$, there exists an initial state x_e when $||x_0 - x_e|| < \delta$ such that $||x(t) - x_e|| > \varepsilon$ for some $t > t_0$.

Asymptotic stability may be defined as follows:

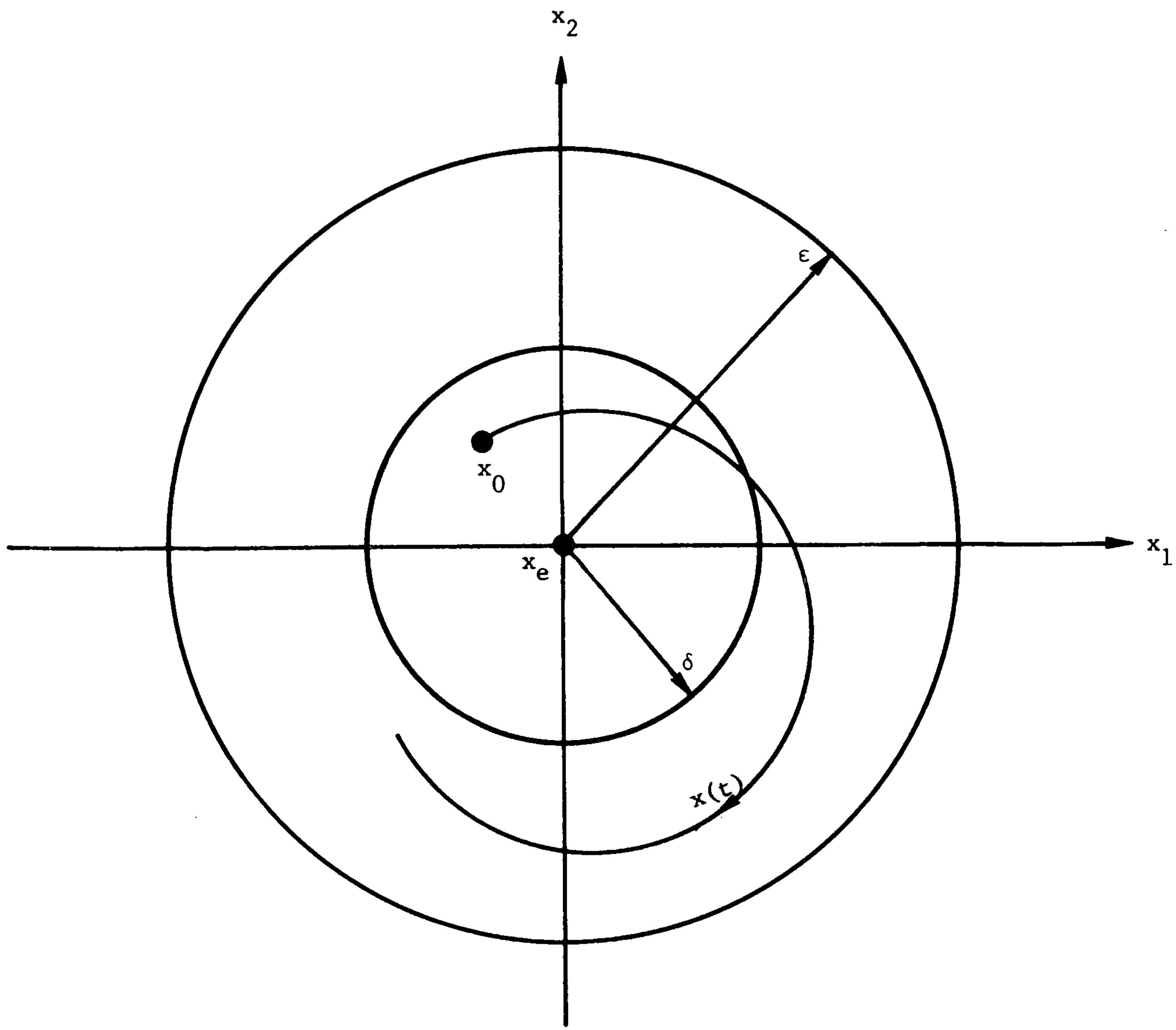

Figure II-1. Illustration of Liapunov stability.

Definition II-3. An equilibrium state, x_e, is asymptotically stable if (1) the equilibrium state is stable, and (2) there exists a $\delta_a > 0$ such that if $||x_0 - x_e|| < \delta_a$, then the system state trajectory approaches x_e as time approaches infinity.

An illustration of asymptotic stability is shown in Figure II-2.

Definition II-4. A system is not asymptotically stable if (1) the equilibrium state is not stable, or (2) if $\delta > 0$, there exists an initial state x_0 where $||x_0 - x_e|| < \delta_a$ such that the system state trajectory does not approach x_e as time approaches infinity.

These concepts of stability are local in nature, i.e., they apply to the stability of a system in a region about the equilibrium state. In addition, the notion of not being Liapunov stable does not imply that the state trajectory will stray arbitrarily far from the equilibrium state.

C. Sign Definiteness of Scalar Functions

For convenience in the discussion of the Liapunov stability theorems in the following section, the concept of sign definiteness of a scalar function will be briefly discussed. Also, a particular class of scalar functions, the quadratic form, will be introduced.

The idea of sign definiteness of a scalar function of an n-dimensional variable x is an extension of the idea of a scalar function of a single variable. For example, a scalar function f(x) is positive on the

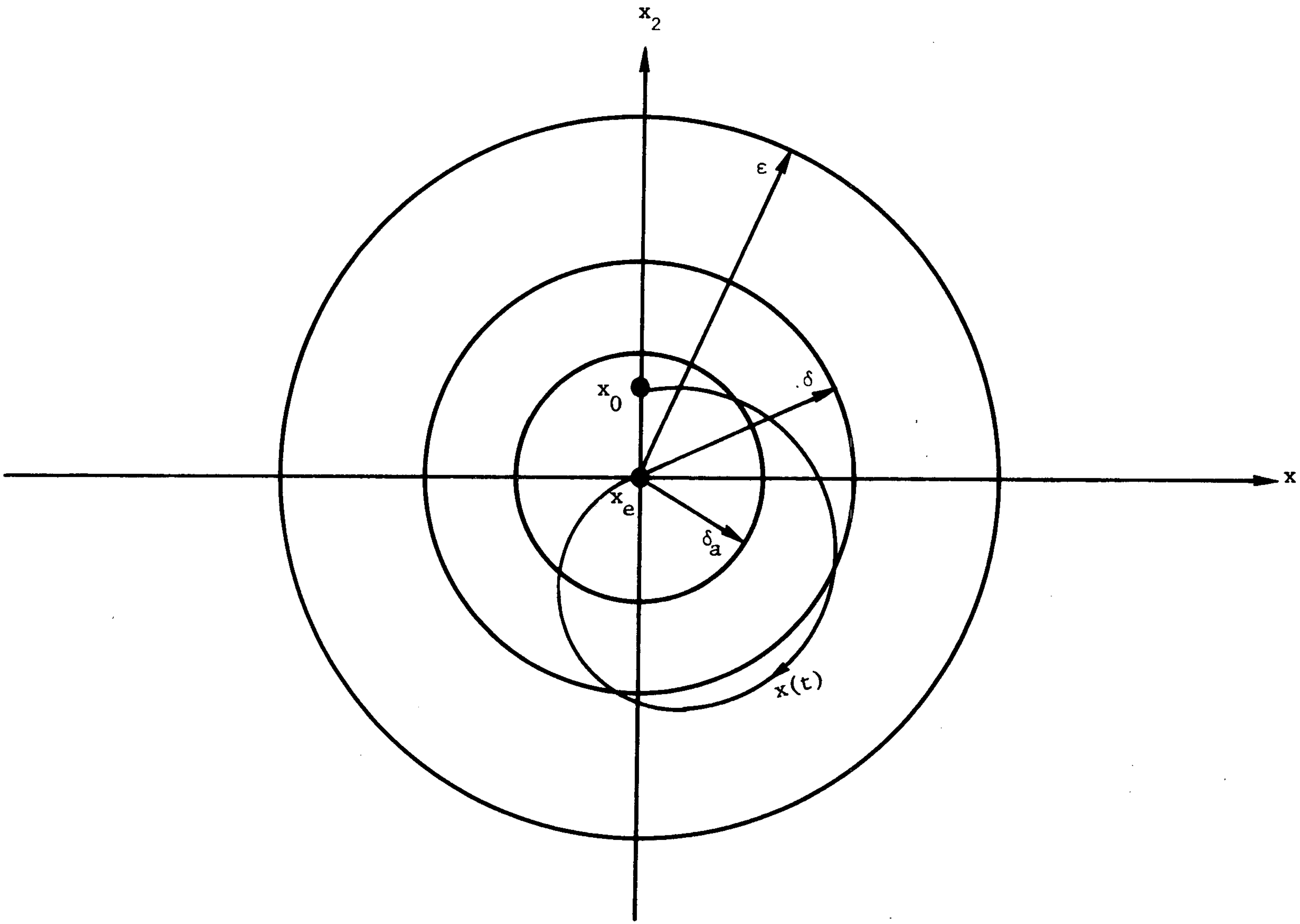

Figure II-2. Illustration of asymptotic stability.

interval $a \leq x \leq b$ if $f(x) > 0$ for all x in this interval. Likewise, a function is negative on the interval $a \leq x \leq b$ if $f(x) < 0$ for all x in this interval. For the case of a scalar function of an n-dimensional variable x, the sign definiteness is dependent on an n-dimensional region.

Definition II-5. A scalar function $V(x)$, is positive definite in the region $||x|| \leq C$ if (1) for all nonzero x in $||x|| \leq C$, $V(x) > 0$, and (2) $V(0) = 0$.

Definition II-6. A scalar function, $V(x)$, is negative definite in the region $||x|| \leq C$ if (1) for all nonzero x in $||x|| \leq C$, $V(x) < 0$, and (2) $V(0) = 0$.

Definition II-7. A scalar function, $V(x)$, is positive-semidefinite in the region $||x|| \leq C$ if (1) for all x in $||x|| \leq C$, $V(x) \geq 0$, and (2) $V(0) = 0$.

Definition II-8. A scalar function, $V(x)$, is negative-semidefinite in the region $||x|| \leq C$ if (1) for all x in $||x|| \leq C$, $V(x) \leq 0$, and (2) $V(0) = 0$.

Definition II-9. A scalar function, $V(x)$, is indefinite if for $C > 0$, there exists a value of x in $||x|| \leq C$ such that $V(x)$ is positive and a value of x is $||x|| \leq C$ such that $V(x)$ is negative.

The next question might be that given a scalar function $V(x)$, how does one use the previous definitions to ascertain the sign definiteness of the scalar function. In general, there is no straight forward approach for determining the sign definiteness of a given scalar function. This fact severely restricts the potential usefulness of the stability theorems of Liapunov. However, there is one particular form of scalar function for which there exists a technique for determining its sign definiteness. Such a scalar function is the quadratic function, which may be written as:

$$V(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} k_{ij} x_i x_j \text{ where}$$

k_{ij}, i, j = 1,..., n are constants. This scalar function involves terms only of second degree in x_i and x_j. The scalar function, V(x) may also be written as:

$$V(x) = x^T Q x, \text{ where}$$

Q is a constant symmetric nxn matrix. A method of determining the sign definiteness of this function is by the use of Sylvester's theorem.

<u>Theorem II-1 - Sylvester's Theorem</u>. A quadratic scalar function, V(x), is positive definite if and only if each of the quantities

$$\det [q_{11}], \quad \det\begin{vmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{vmatrix}, \quad \det\begin{vmatrix} q_{11} & q_{12} & q_{13} \\ q_{12} & q_{22} & q_{23} \\ q_{13} & q_{23} & q_{33} \end{vmatrix}, \ldots, \det [Q]$$

are greater than zero.

If any of the above determinants fails to be positive by being equal to zero, then the scalar function is positive semidefinite. A matrix Q is negative definite or negative semidefinite if the matrix -Q is positive definite or positive semidefinite, respectively.

D. Liapunov Stability Theorems.

The basis of Liapunov's direct method may be considered as a generalization of the energy concepts for mechanical systems. For example, a mechanical system is stable and tending towards an equilibrium state if the total energy of the system is decreasing. This basic idea, extended to n-dimensions, may be used to determine if an equilibrium state of a system is asymptotically stable, stable in the sense of Liapunov or unstable. Proofs of these theorems are given in [10].

Theorem II-2 - Liapunov's First Stability Theorem. An equilibrium state, x_e, of a system is stable in the sense of Liapunov if (1) there exists a scalar function, V(x), which is continuous and has continuous first partial derivatives in some region R about the equilibrium state, (2) V(x) is positive definite in the region R and, (3) the time derivative of the scalar function, $\dot{V}(x)$, evaluated along the trajectories of the system under investigation, is negative semidefinite in the region R.

Theorem II-3 - Liapunov's Second Stability Theorem. An equilibrium state, x_e, of a system is asymptotically stable if (1) there exists a scalar function, $V(x)$, which is continuous and has continuous first partial derivatives in some region R about the equilibrium state, (2) $V(x)$ is positive definite in the region R, and (3) the time derivative of the scalar function, $\dot{V}(x)$, evaluated along the trajectories of the system under investigation, is negative definite in the region R.

Theorem II-4. An equilibrium state, x_e, of a system is asymptotically stable if (1) the equilibrium state is Liapunov stable and (2) the curve $\dot{V} = 0$ is not a system trajectory.

The preceding theorems provide only sufficient conditions for determining the stability of an equilibrium state. Thus, for a given system, if a scalar function that satisfies the conditions of the stability theorems cannot be found, it does not mean that the equilibrium state is unstable. We have only failed to establish the stability of the equilibrium state. The following theorems may be used to establish the instability of an equilibrium state.

Theorem II-5 - Liapunov's First Instability Theorem. An equilibrium state, x_e, of a system is unstable if (1) there exists a scalar function, $V(x)$, which is continuous and has continuous first partial derivatives in some region R about the equilibrium state, (2) the time derivative of the scalar function, $\dot{V}(x)$, evaluated along the trajectories of the system under investigation, is positive definite, and (3) the scalar function $V(x)$ is indefinite, positive semidefinite, or positive definite in a region, R, arbitrarily near the equilibrium state.

Theorem II-6 - Liapunov's Second Instability Theorem. An equilibrium state of a system, x_e, is unstable if (1) there exists a scalar function, $V(x)$, which is continuous and has continuous first partial derivatives in some region R about the equilibrium state, (2) the time derivative of the scalar function, $\dot{V}(x)$, evaluated along the trajectories of the system under investigation, is of the form

$$\dot{V}(x) = \lambda V(x) + V_1(x) \text{ where}$$

$\lambda > 0$ and $V_1(x)$ is positive semidefinite in the region R, and (3) the scalar function $V(x)$ is indefinite, positive semidefinite, or positive definite in the region, R, arbitrarily near the equilibrium state.

In order to illustrate the use of Liapunov's stability theorems, let us consider the stability of the equilibrium states of Van der Pol's equation.

$$\frac{d^2x}{dt^2} + \mu(x^2-1)\frac{dx}{dt} + x = 0 \qquad \text{(II-1)}$$

where μ is an arbitrary constant. In state equation form, this may be written as:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 - \mu(x_1^2 - 1)x_2 \tag{II-2}$$

The only equilibrium state of the system is obviously, $x_1 = x_2 = 0$. Investigation of equation (II-2) shows that for $\mu > 0$ a stable limit cycle is indicated. This is, indeed, the case. The limit-cycle boundaries are shown in Figure II-3 for a value of $\mu = 1$. For $\mu < 0$, an unstable limit cycle is indicated and this is exemplified in Figure II-4 by two system trajectories, one inside the limit-cycle boundaries and one outside. Thus, for $\mu > 0$, the equilibrium state is unstable in the sense of Liapunov and for $\mu < 0$, the equilibrium state is asymptotically stable.

In order to show this using Liapunov's stability theorems, consider first the case $\mu = 1$. The state equations may be written as:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 - (x_1^2 - 1)x_2 \tag{II-3}$$

As a scalar function, V(x), select

$$V(x) = x_1^2(3/2 - 1/4x_1^2) + x_2^2 - x_1x_2 \tag{II-4}$$

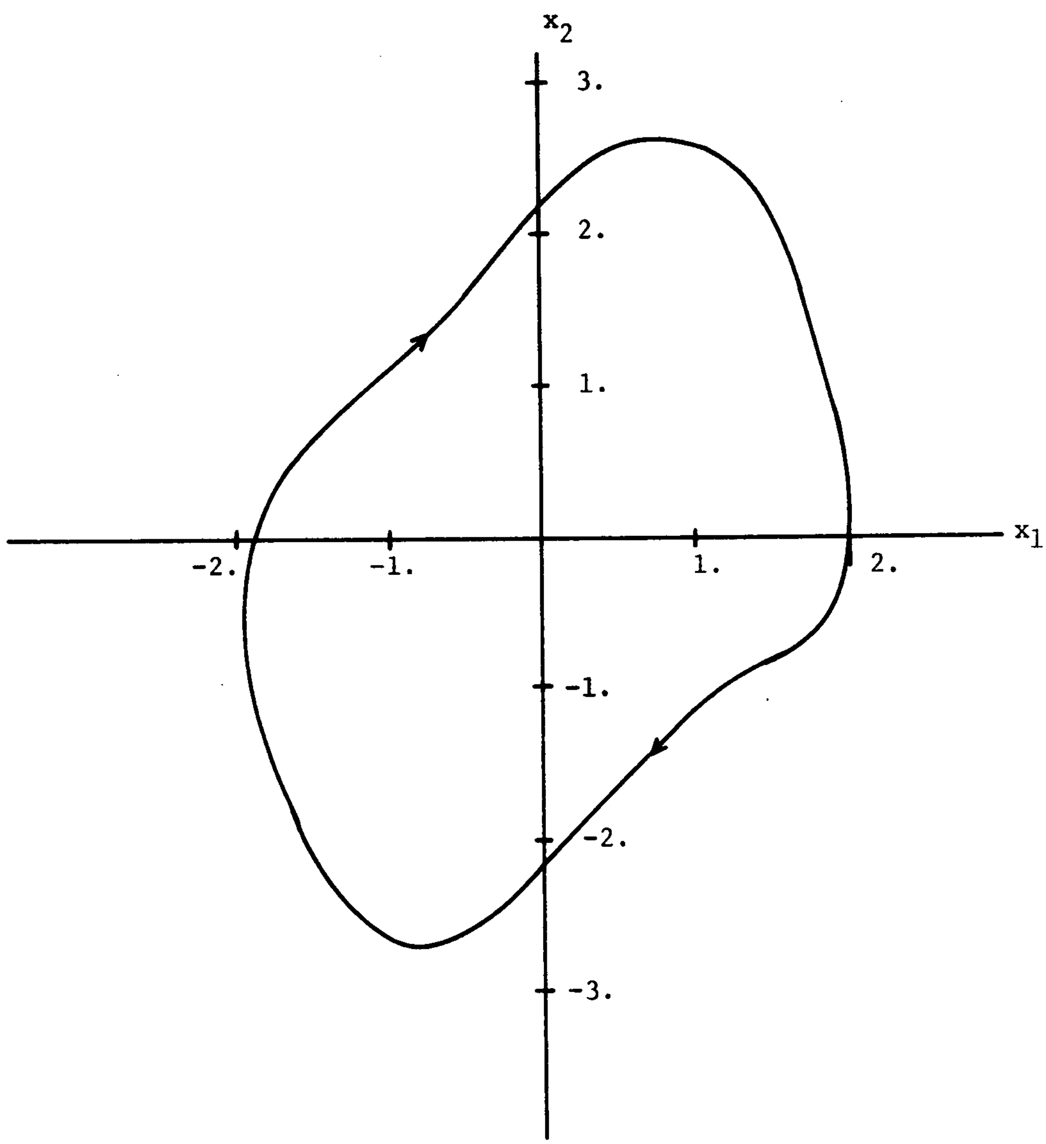

Figure II-3. Limit cycle boundaries for Van der Pol's equation.

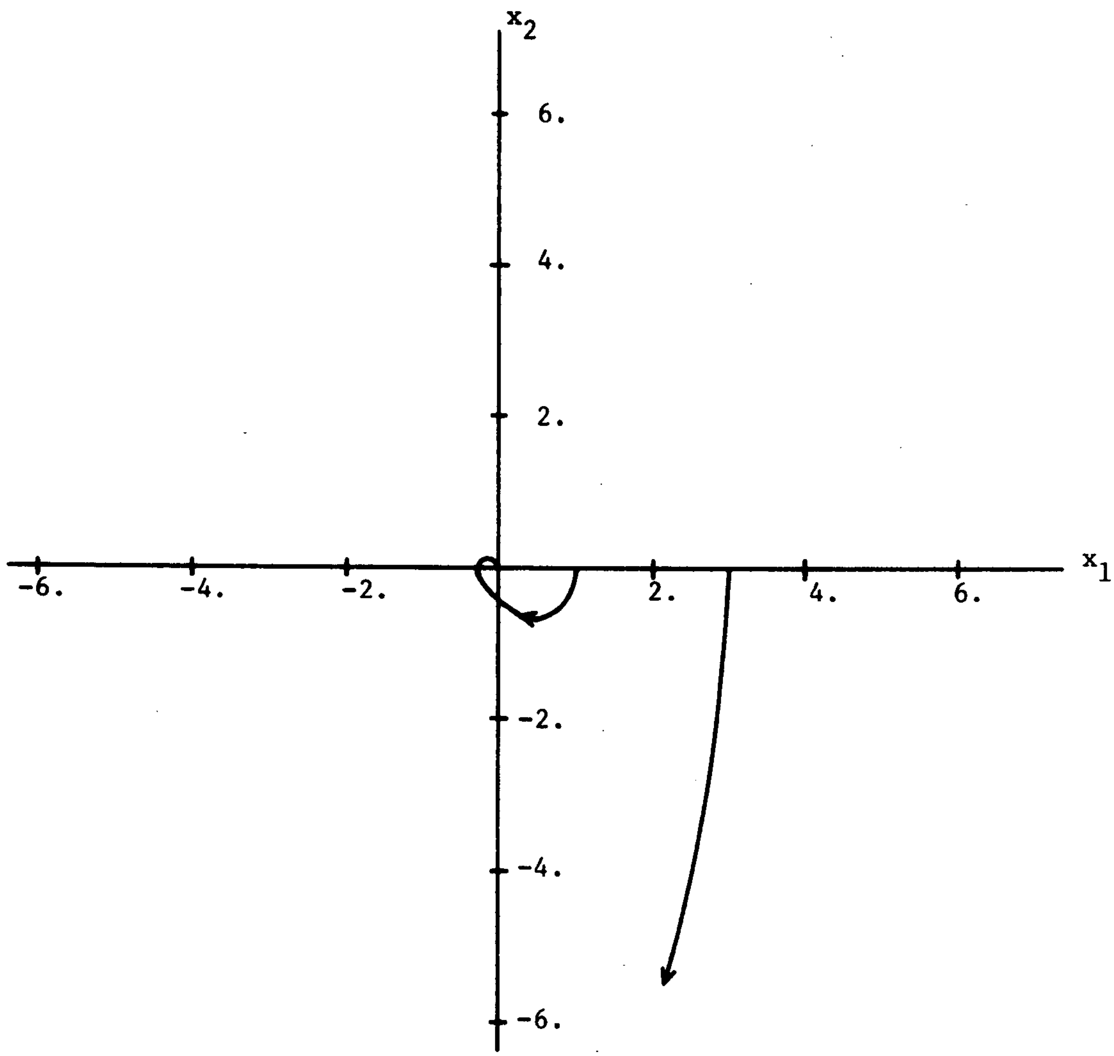

Figure II-4. Illustration of unstable limit cycle for Van der Pol's equation.

This function is positive definite for $x_1^2 < 5$. The time derivative of equation II-4 evaluated along the system trajectories is:

$$\dot{V}(x) = x_1^2 + x_2^2(1-2x_1^2) \tag{II-5}$$

Equation II-5 is positive definite for x_1^2 less than 1/2. Therefore, from Theorem II-5, the equilibrium state is unstable.

For the case $\mu = -1$, the state equations may be written as:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 + (x_1^2-1)x_2 \tag{II-6}$$

Again, the single equilibrium state of the system is $x_1 = x_2 = 0$. For this case, select V(x) to be:

$$V(x) = 1/2\ x_1^2 + x_2^2 + 1/4x_1^4 \tag{II-7}$$

$\dot{V}(x)$, evaluated along the system trajectories is given by:

$$\dot{V}(x) = -x_1^2 - 3x_2^2 + 2x_1^2x_2^2 \tag{II-8}$$

Thus, the conditions of Theorem II-3 are satisfied for x_1^2 less than 3/2 and the equilibrium state is asymptotically stable.

E. Application of Liapunov's Direct Method to Model-Reference Adaptive Control Systems.

The basic idea in the application of Liapunov's direct method for the design of model-reference adaptive control systems is to design the system adaption in such a manner that the response error (model output minus plant output) will be asymptotically stable. If by using Liapunov's direct method it can be shown that the response error is asymptotically stable, then it follows that the plant will track the model.

The response error, in equation form, is given by:

$$e(t) = x_m(t) - x_p(t) \tag{II-9}$$

If the adaption is designed so that the response error is asymptotically stable and the model is assumed asymptotically stable, then

$$\lim_{t \to \infty} e(t) = \lim_{t \to \infty} x_m(t) - \lim_{t \to \infty} x_p(t) \tag{II-10}$$

Since $\lim_{t \to \infty} e(t) = \lim_{t \to \infty} x_m(t) = 0$, then

$$\lim_{t \to \infty} x_p(t) = 0 \tag{II-11}$$

and the plant will adapt so as to track the model.

III. TIME-DOMAIN MODEL REFERENCE ADAPTIVE CONTROL SYSTEM DESIGN

This chapter presents the results of the present research in the area of time-domain design of model-reference adaptive control systems using Liapunov's direct method. The time-varying model-reference adaptive problem is formulated precisely and the adaption technique is derived analytically. The inclusion of several positive semidefinite terms into a Liapunov function results in an adaptation rule which is a function of the response error, the input, the plant states, and the derivative and integral of these quantities. The design technique, as well as being applicable to a class of time-varying systems is also shown to be applicable, with certain assumptions, to a class of non-linear adaptive systems. Simulation results of a model of the National Aeronautics and Space Administration's Space Shuttle vehicle show that the adaptation technique provides good convergence for the adaptive parameters and that the response error is greatly reduced as compared to the non-adapted system.

A. Problem Formulation

The time-varying adaptive control problem considered is assumed to be described by the state equations:

$$\underline{\dot{x}}_p(t) = A_p(t)\underline{x}_p(t) + B_p(t)\underline{u}(t) \tag{III-1}$$

where $\underline{x}_p(t)$ is an n-vector, $\underline{u}(t)$ is an r-vector, and $A_p(t)$ and $B_p(t)$ are n x n and n x r matrices, respectively. The elements of $A_p(t)$ and $B_p(t)$ consist, in general, of unknown, time-varying adjustable parameters. Each element of the A_p and B_p matrices is assumed to be of the form:

$$\begin{aligned} a^p_{ij}(t) &= c^a_{ij}(t) + K^a_{ij}(t) \\ b^p_{ij}(t) &= c^b_{ij}(t) + K^b_{ij}(t) \end{aligned} \tag{III-2}$$

where $K^a_{ij}(t)$ and $K^b_{ij}(t)$ are the adaption parameters, and in general, $c^a_{ij}(t)$ and $c^b_{ij}(t)$ are time varying unknown values. One basic assumption, which is necessary for mathematical rigor, will now be made concerning the elements of the A_p and B_p matrices. The assumption is that the time rate of change of the unknown portion of the plant parameters, i.e., $\dot{c}^a_{ij}(t)$ and $\dot{c}^b_{ij}(t)$, may be considered negligible as compared to the time rate of change of the adaption parameters, $\dot{K}^a_{ij}(t)$ and $\dot{K}^b_{ij}(t)$.

The model control system is assumed to be of the form:

$$\underline{x}_m(t) = A_m\underline{x}_m(t) + B_m\underline{u}(t) \tag{III-3}$$

where $\underline{x}_m(t)$ is an n-vector, $\underline{u}$ is an r-vector, and A_m and B_m are n x n and n x r constant matrices, respectively.

This is basically the same system as in [13] and [14] except a time-varying plant is considered here. The response error in the model-reference scheme is given by:

$$\underline{e}(t) \overset{\Delta}{=} \underline{x}_m(t) - \underline{x}_p(t) \qquad \text{(III-4)}$$

The error state equation may then be derived from Equations (III-1), (III-3), and (III-4) as:

$$\dot{\underline{e}}(t) = A_m\underline{e}(t) + A(t)\underline{x}_p(t) + B(t)\underline{u}(t) \qquad \text{(III-5)}$$

where $A(t) = A_m - A_p(t)$ and $B(t) = B_m - B_p(t)$.

The reference model output, $\underline{x}_m(t)$, corresponds to the desired output of the adaptive control system when subjected to the input $\underline{u}(t)$. The design objective then is to adjust the parameters $K_{ij}^{a}(t)$ and $K_{ij}^{b}(t)$ so that $\underline{x}_p(t)$ closely approximates $\underline{x}_m(t)$ regardless of plant parameter variations.

B. Derivation of Adaption Rule

The first step in the derivation of the adaption rule is to select a Liapunov function which is a quadratic form of the error plus several positive semidefinite terms. The positive semidefinite terms will be a function of the error states, the plant states, the inputs, and the adaption parameters. For convenience in notation, the dependence on time of the various quantities will be denoted explicitly.

The Liapunov function selected is:

$$V = e^T Qe + \sum_{i,j=1}^{n} \frac{1}{\alpha_{ij}} \{a_{ij} + \beta_{ij} \sum_{k=1}^{n} e_k q_{ki} x_{pj} +$$

$$\rho_{ij} \frac{d}{dt}\left[\sum_{k=1}^{n} e_k q_{ki} x_{pj}\right]\}^2 + \sum_{i,j=1}^{n} \rho_{ij}\left[\sum_{k=1}^{n} e_k q_{ki} x_{pj}\right]^2$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{r} \frac{1}{\gamma_{ij}} \{b_{ij} + \delta_{ij} \sum_{k=1}^{n} e_k q_{ki} u_j$$

$$+ \sigma_{ij} \frac{d}{dt}\left[\sum_{k=1}^{n} e_k q_{ki} u_j\right]\}^2 + \sum_{i=1}^{n}\sum_{j=1}^{r} \sigma_{ij}\left[\sum_{k=1}^{n} e_k q_{ki} u_j\right]^2 \qquad \text{(III-6)}$$

Q is a symmetric positive-definite matrix which will be determined later, a_{ij} and b_{ij} are elements of the A and B matrices, respectively, α_{ij} and γ_{ij} are constants > 0, and β_{ij}, ρ_{ij}, δ_{ij}, and σ_{ij} are constants ≥ 0. This Liapunov function differs considerably from that of [14] as the basic form has been modified by the addition of the terms and cross-product terms associated with the coefficients ρ_{ij} and σ_{ij}. With the inclusion of these terms, the adaption parameters become a function of the integral and the derivative of the error as well as the error itself. The time derivative of Equation (III-6), term by term, is

$$\dot{V} = \underline{e}^T [A_m Q + QA_m]\underline{e} \quad + \quad 2\underline{e}^T QA\underline{x}_p \; + \; 2\underline{e}^T QB\underline{u}$$

$$+ \; 2 \sum_{i,j=1}^{n} \frac{a_{ij}\dot{a}_{ij}}{\alpha_{ij}}$$

$$+ 2 \sum_{i,j=1}^{n} \frac{\beta_{ij} \dot{a}_{ij}}{\alpha_{ij}} \sum_{k=1}^{n} e_k q_{ki} x_{pj}$$

$$+ 2 \sum_{i,j=1}^{n} \frac{\beta_{ij}}{\alpha_{ij}} a_{ij} \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right]$$

$$+ 2 \sum_{i,j=1}^{n} \frac{\rho_{ij}}{\alpha_{ij}} \dot{a}_{ij} \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right]$$

$$+ 2 \sum_{i,j=1}^{n} \frac{\rho_{ij}}{\alpha_{ij}} a_{ij} \frac{d^2}{dt^2} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right]$$

$$+ 2 \sum_{i,j=1}^{n} \frac{\beta_{ij} \rho_{ij}}{\alpha_{ij}} \left\{ \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right] \right\}^2$$

$$+ 2 \sum_{i,j=1}^{n} \frac{\beta_{ij} \rho_{ij}}{\alpha_{ij}} \sum_{k=1}^{n} e_k q_{ki} x_{pj} \frac{d^2}{dt^2} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right]$$

$$+ 2 \sum_{1,j=1}^{n} \frac{\beta_{ij}^2}{\alpha_{ij}} \sum_{k=1}^{n} e_k q_{ki} x_{pj} \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right]$$

$$+ 2 \sum_{1,j=1}^{n} \frac{\rho_{ij}^2}{\alpha_{ij}} \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right] \frac{d^2}{dt^2} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right]$$

$$+ 2 \sum_{1,j=1}^{n} \rho_{ij} \sum_{k=1}^{n} e_k q_{ki} x_{pj} \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right]$$

$$+ 2 \sum_{i=1}^{n} \sum_{j=1}^{r} \frac{b_{ij} \dot{b}_{ij}}{\gamma_{ij}}$$

$$+ 2 \sum_{i=1}^{n} \sum_{j=1}^{r} \frac{\delta_{ij} \dot{b}_{ij}}{\gamma_{ij}} \sum_{k=1}^{n} e_k q_{ki} u_j$$

$$+ 2 \sum_{i=1}^{n} \sum_{j=1}^{r} \frac{\delta_{ij} b_{ij}}{\gamma_{ij}} \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} u_j \right]$$

$$+ 2 \sum_{i=1}^{n} \sum_{j=1}^{r} \frac{\delta_{ij} \dot{b}_{ij}}{\gamma_{ij}} \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} u_j \right]$$

$$+ 2 \sum_{i=1}^{n} \sum_{j=1}^{r} \frac{\sigma_{ij} b_{ij}}{\gamma_{ij}} \frac{d^2}{dt^2} \left[\sum_{k=1}^{n} e_k q_{ki} u_j \right]$$

$$+ 2 \sum_{i=1}^{n} \sum_{j=1}^{r} \frac{\delta_{ij} \sigma_{ij}}{\gamma_{ij}} \{ \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} u_j \right] \}^2$$

$$+ 2 \sum_{i=1}^{n} \sum_{j=1}^{r} \frac{\delta_{ij} \sigma_{ij}}{\gamma_{ij}} \sum_{k=1}^{n} e_k q_{ki} u_j \frac{d^2}{dt^2} \left[\sum_{k=1}^{n} e_k q_{ki} u_j \right]$$

$$+ 2 \sum_{i=1}^{n} \sum_{j=1}^{r} \frac{\delta_{ij}^2}{\gamma_{ij}} \sum_{k=1}^{n} e_k q_{ki} u_j \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} u_j \right]$$

$$+ 2 \sum_{i=1}^{n} \sum_{j=1}^{r} \frac{\sigma_{ij}^2}{\gamma_{ij}} \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} u_j \right] \frac{d}{dt^2} \left[\sum_{k=1}^{n} e_k q_{ki} u_j \right]$$

$$+ 2 \sum_{i=1}^{n} \sum_{j=1}^{r} \sigma_{ij} \sum_{k=1}^{n} e_k q_{ki} u_j \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} u_j \right] \qquad \text{(III-7)}$$

In order to prove that the response error is asymptotically stable, it is necessary to choose $\dot{a}_{ij}$ and $\dot{b}_{ij}$ so that the time derivative of the Liapunov function is negative definite. Now,

$$e^T Q A x_p =$$

$$(e_1 q_{11} + e_2 q_{21} + \cdots + e_n q_{n1}) \cdot (a_{11} x_{p1} + a_{12} x_{p2} + \cdots + a_{1n} x_{pn}) +$$

$$(e_1 q_{12} + e_2 q_{22} + \cdots + e_n q_{2n}) \cdot (a_{21} x_{p1} + a_{22} x_{pn} + \cdots + a_{2n} x_{pn}) + \cdots$$

$$\vdots \qquad\qquad \vdots$$

$$+ (e_1 q_{1n} + e_2 q_{2n} + \cdots + e_n q_{nn}) \cdot (a_{n1} x_{p1} + a_{n2} x_{p2} + \cdots + a_{nn} x_{pn})$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} a_{ij} e_k q_{ki} x_{pj}$$

and

$$e^T QBu =$$

$$(e_1 q_{11} + e_2 q_{21} + \cdots + e_n q_{n1}) \cdot (b_{11} u_1 + b_{12} u_2 + \cdots + b_{1r} u_r) +$$

$$(e_1 q_{12} + e_2 q_{22} + \cdots + e_n q_{n2}) \cdot (b_{21} u_1 + b_{22} u_2 + \cdots + b_{2r} u_r) + \cdots$$

$$\vdots \qquad \qquad \vdots$$

$$+ (e_1 q_{1n} + e_2 q_{2n} + \cdots + e_n q_{nn}) \cdot (b_{n1} u_1 + b_{n2} u_2 + \cdots + b_{nr} u_r)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{r} \sum_{k=1}^{n} b_{ij} e_k q_{ki} u_j$$

Let the parameters $\dot{a}_{ij}$ and $\dot{b}_{ij}$, which are directly related to the adaption parameters, be of the form:

$$\dot{a}_{ij} = -\alpha_{ij} \sum_{k=1}^{n} e_k q_{ki} x_{pj} - \beta_{ij} \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right]$$

$$- \rho_{ij} \frac{d^2}{dt^2} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right] \qquad \text{(III-8)}$$

$$\dot{b}_{ij} = -\gamma_{ij} \sum_{k=1}^{n} e_k q_{ki} u_j - \delta_{ij} \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right]$$

$$- \sigma_{ij} \frac{d^2}{dt^2} \left[\sum_{k=1}^{n} e_k q_{ki} u_j \right] \tag{III-9}$$

Substituting the above values for $\dot{a}_{ij}$ and $\dot{b}_{ij}$ into Equation (III-7), and collecting terms, $\dot{V}$ becomes:

$$\dot{V} = e^T(A_m^T Q + QA_m)e - 2 \sum_{i,j=1}^{n} \beta_{ij} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj} \right]^2$$

$$-2 \sum_{i=1}^{n} \sum_{j=1}^{r} \delta_{ij} \left[\sum_{k=1}^{n} e_k q_{ki} u_j \right]^2 \tag{III-10}$$

This cancellation of terms in the expression of $\dot{V}$ by the substitution of $\dot{a}_{ij}$ and $\dot{b}_{ij}$ is accomplished as exemplified below. The term $2 \sum_{i,j=1}^{n} \frac{a_{ij} \dot{a}_{ij}}{\alpha_{ij}}$ upon substitution of $\dot{a}_{ij}$ from Equation (III-8) becomes:

$$-2 \sum_{i,j=1}^{n} a_{ij} \sum_{k=1}^{n} e_k q_{ki} x_{pj}$$

$$-2 \sum_{i,j=1}^{n} \frac{a_{ij}\beta_{ij}}{\alpha_{ij}} \frac{d}{dt} \sum_{k=1}^{n} e_k q_{ki} x_{pj}$$

$$-2 \sum_{i,j=1}^{n} \frac{a_{ij}\rho_{ij}}{\alpha_{ij}} \frac{d^2}{dt^2} \sum_{k=1}^{n} e_k q_{ki} x_{pj}$$

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Thus, these terms will cancel with terms 2, 6, and 8 of Equation (III-7). The remaining terms may be eliminated in a similar manner. Since, by assumption, β_{ij} and δ_{ij} are greater than or equal 0, it is necessary only to show that the first term of Equation (III-10) is negative definite in order to prove that the response error is asymptotically stable.

It is known [10] that if the matrix A_m is a stable matrix, there exists a unique symmetric positive-definite matrix Q, which satisfies the equation

$$A_m^T Q + QA_m = -C \qquad \text{(III-11)}$$

where C is any symmetric positive-definite matrix. Thus, for any positive-definite C, V is a positive-definite quadratic function of the reponse error and $\dot{V}$ is a negative-definite quadratic function of the response error. Therefore, from Theorem II-3, the error is asymptotically stable and the plant output will track the model output.

The adaption parameters rates, $\dot{K}_{ij}^a$ and $\dot{K}_{ij}^b$ may be determined from the relationships

$$a_{ij} = a_{ij}^m - a_{ij}^p = a_{ij}^m - c_{ij}^a(t) - K_{ij}^a(t)$$

$$b_{ij} = b_{ij}^m - b_{ij}^p = b_{ij}^m - c_{ij}^b(t) - K_{ij}^b(t)$$

Taking the time derivative of a_{ij} and b_{ij} and invoking the assumption that $\dot{c}^a_{ij}(t)$ and $\dot{c}^b_{ij}(t)$ are negligible as compared to $\dot{K}^a_{ij}(t)$ and $\dot{K}^b_{ij}(t)$, respectively, the adaptive parameter rates are given by the expression:

$$\dot{K}^a_{ij}(t) = -\dot{a}_{ij}(t)$$

$$\dot{K}^b_{ij}(t) = -\dot{b}_{ij}(t) \qquad \text{(III-12)}$$

Integrating Equation 12, the adaption parameters are:

$$K^a_{ij} = \alpha_{ij} \int_{t_o}^{t} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj}\right] d\phi + \beta_{ij} \sum_{k=1}^{n} e_k q_{ki} x_{pj}$$
$$+ \rho_{ij} \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} x_{pj}\right] + K^a_{ij}(t_o) \qquad \text{(III-13)}$$

$$K^b_{ij} = \gamma_{ij} \int_{t_o}^{t} \left[\sum_{k=1}^{n} e_k q_{ki} u_j\right] d\phi + \delta_{ij} \sum e_k q_{ki} u_j$$
$$+ \sigma_{ij} \frac{d}{dt} \left[\sum_{k=1}^{n} e_k q_{ki} u_j\right] + K_{ij}(t_o) \qquad \text{(III-14)}$$

Equations (III-13) and (III-14) then give the general relationships for the adaptive plant parameters.

C. Application of Design Technique

In order to illustrate the application of the design technique, a model of the National Aeronautics and Space Administration's Space Shuttle vehicle is used. The design technique is applied first to a model with linear characteristics and then to a model with nonlinear characteristics. A linear simulation study was necessary since by using modulation techniques, virtually linear characteristics in the Space Shuttle vehicle may be obtained [15]. Simulation studies were also made in order to determine the effect on the adaptation caused by wind gust disturbances on the system. The dynamic equations used for the Space Shuttle vehicle is assumed to have one bending mode in addition to the rotational mode. In transfer function notation, the dynamic equations for the plant and model may be represented as:

$$\frac{\eta_p(s)}{U(s)} = \frac{s}{s^2 + .2s + 4}$$

$$\frac{\phi_p(s)}{U(s)} = \frac{1}{s^2}$$

$$\frac{\eta_m(s)}{U(s)} = \frac{s}{s^2 + 2s + 4}$$

$$\frac{\phi_m(s)}{U(s)} = \frac{1}{s^2 + 1.414s + 1} \qquad \text{(III-15)}$$

In state equation form, the plant equations may be represented as:

$$\dot{x}_{1p} = x_{2p}$$

$$\dot{x}_{2p} = -4.x_{1p} - .2x_{2p} + u$$

$$\dot{x}_{3p} = x_{4p} + u$$

$$\dot{x}_{4p} = u \qquad \text{(III-16)}$$

where $x_{1p} = \eta$, $x_{2p} = \dot{\eta}$, $x_{3p} = \phi + \dot{\phi}$, and $x_{4p} = \dot{\phi}$. The matrices $A_p(t)$ and $B_p(t)$, from Equation (III-1) are

$$A_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -4. & -.2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and}$$

$$B = [0 \quad 1 \quad 1 \quad 1]^T$$

For simulation purposes, it is necessary to assume specific values for the elements of the A_p and B_p matrices. The design technique does not, however, require the value of these elements for its implementation. The rotational mode consists only of an inertial load and the bending mode has a damping ratio of .05 with an undamped natural frequency of 2. rad/sec. It is assumed that all states are available.

The model response chosen for the adapted plant in state equation form is:

$$\dot{x}_{1m} = x_{2m}$$

$$\dot{x}_{2m} = -4.x_{1m} - 2.x_{2m} + u$$

$$x_{3m} = -x_{3m} + .586x_{4m} + u$$

$$x_{4m} = -x_{3m} - .414x_{4m} + u \tag{III-17}$$

The matrices A_m and B_m of Equation (III-3) are:

$$A_m = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -4. & -2. & 0 & 0 \\ 0 & 0 & -1. & .586 \\ 0 & 0 & -1. & -.414 \end{bmatrix} \quad \text{and}$$

$$B_m = [0 \quad 1 \quad 1 \quad 1]^T$$

For the bending mode characteristics, the model has a damping ratio of .5 and an undamped natural frequency of 2. rad/sec. The rotational mode model has a damping ratio of .707 and an undamped natural frequency of 1 rad/sec. The parameters selected for adaption are the damping ratio of the bending mode and the damping ratio and natural frequency of the rotational mode.

In Equation (III-11), a solution for the matrix Q may be obtained with the selection of a positive-definite matrix C.

Selecting the C matrix as:

$$C = \begin{bmatrix} 8. & 0. & 0. & 0. \\ 0. & 2. & 0. & 0. \\ 0. & 0. & 8. & 1.656 \\ 0. & 0. & 1.656 & .484 \end{bmatrix}$$

The Q matrix may be determined as:

$$Q = \begin{bmatrix} 6. & 1. & 0 & 0 \\ 1. & 1. & 0 & 0 \\ 0 & 0 & 3. & 1. \\ 0 & 0 & 1. & 2. \end{bmatrix}$$

The adaptive parameters, which from practical considerations are assumed to be zero at time zero, from Equation (III-13) are:

$$K^a_{22} = \alpha_{22} \int_0^t (e_1 x_{p2} + e_2 x_{p2})\, d\tau$$

$$+ \beta_{22}(e_1 x_{p2} + e_2 x_{p2}) + \rho_{22}\frac{d}{dt}(e_1 x_{p2} + e_2 x_{p2}) \qquad \text{(III-18)}$$

$$K^a_{33} = \alpha_{33} \int_0^t (3.e_3 x_{p3} + e_4 x_{p3})\, d\tau$$

$$+ \beta_{33}(3.e_3 x_{p3} + e_4 x_{p3}) + \rho_{33}\frac{d}{dt}(3.e_3 x_{p3} + e_4 x_{p3}) \qquad \text{(III-19)}$$

$$K^{a}_{34} = \alpha_{34} \int_0^t (3.e_3x_{p4}+e_4x_{p4})d\pi$$

$$+ \beta_{34}(3.e_3x_{p4}+e_4x_{p4}) + \rho_{34}\frac{d}{dt}(3.e_3x_{p4}+e_4x_{p4}) \qquad \text{(III-20)}$$

$$K^{a}_{43} = \alpha_{43} \int_0^t (e_3x_{p3}+2.e_4x_{p3})d\pi$$

$$+ \beta_{43}(e_3x_{p3}+2.e_4x_{p3}) + \rho_{43}\frac{d}{dt}(e_3x_{p3}+2.e_4x_{p3}) \qquad \text{(III-21)}$$

$$K^{a}_{44} = \alpha_{44} \int_0^t (e_3x_{p4}+2.e_4x_{p4})d\pi$$

$$+ \beta_{44}(e_3x_{p4}+2.e_4x_{p4}) + \rho_{44}\frac{d}{dt}(e_3x_{p4}+2.e_4x_{p4}) \qquad \text{(III-22)}$$

These adaption equations were implemented in a digital computer program utilizing the Continuous System Modeling Program (CSMP). For the linear case, two system inputs were investigated, the first being a sine wave and the second a step. Figure III-1 shows the bending response of the unadapted plant, the adapted plant, and the model response for a sine wave input. The gains α_{22}, β_{22}, and ρ_{22} are 10., 5., and 5., respectively. Figure III-2 shows the bending response for the unadapted plant, the adapted plant, and the model with a step input to the system. The gains α_{22}, β_{22}, and ρ_{22} are the same as in the previous case. Figure III-3 shows a plot of the response error for a sine wave input to the system for 2 values of ρ_{22}. Figure III-3 illustrates that the derivative term of Equation (III-17) causes a significant reduction in the maximum

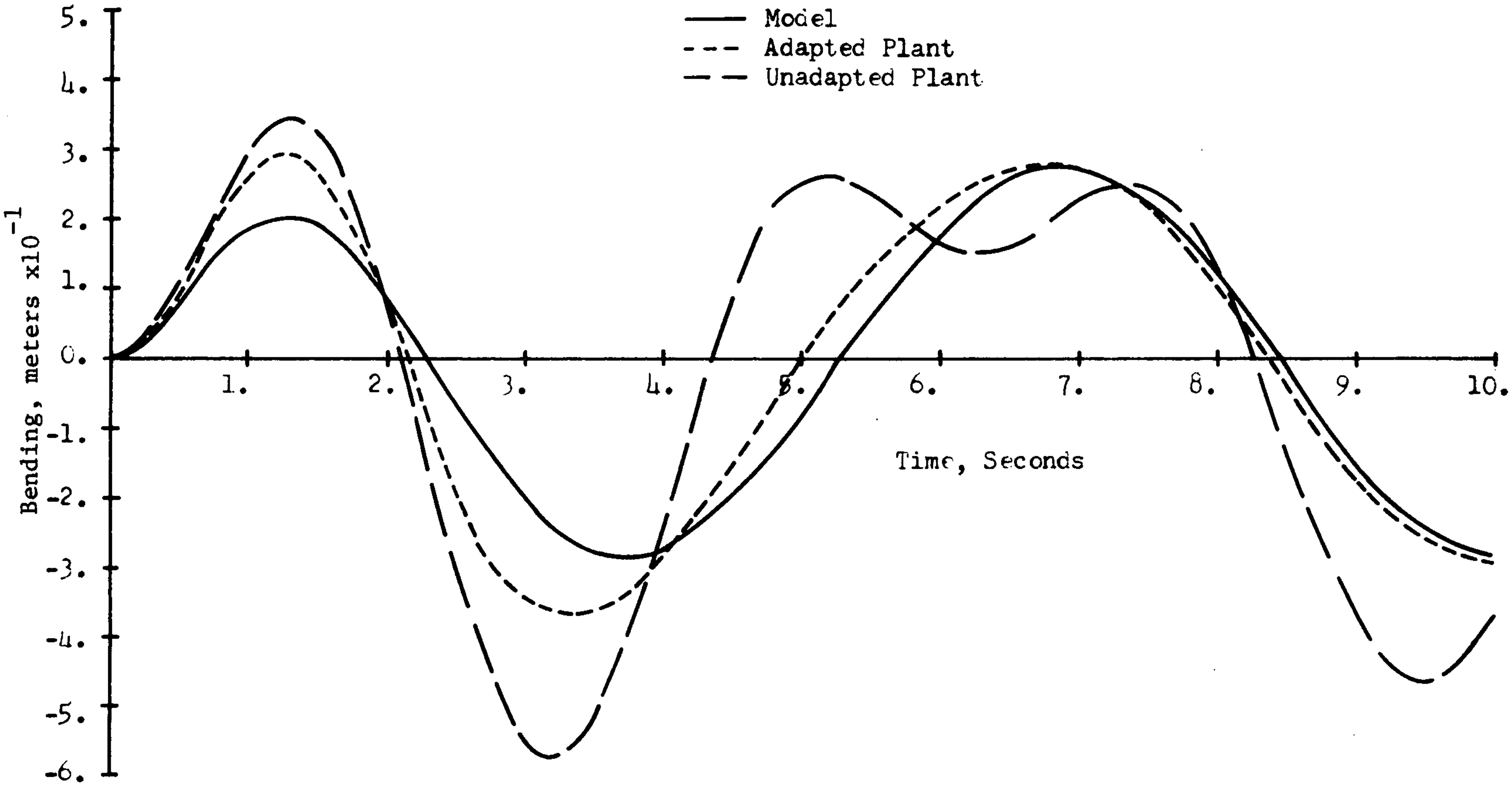


Figure III-1. Bending mode responses of model, adapted plant, and unadapted plant, sine wave input.

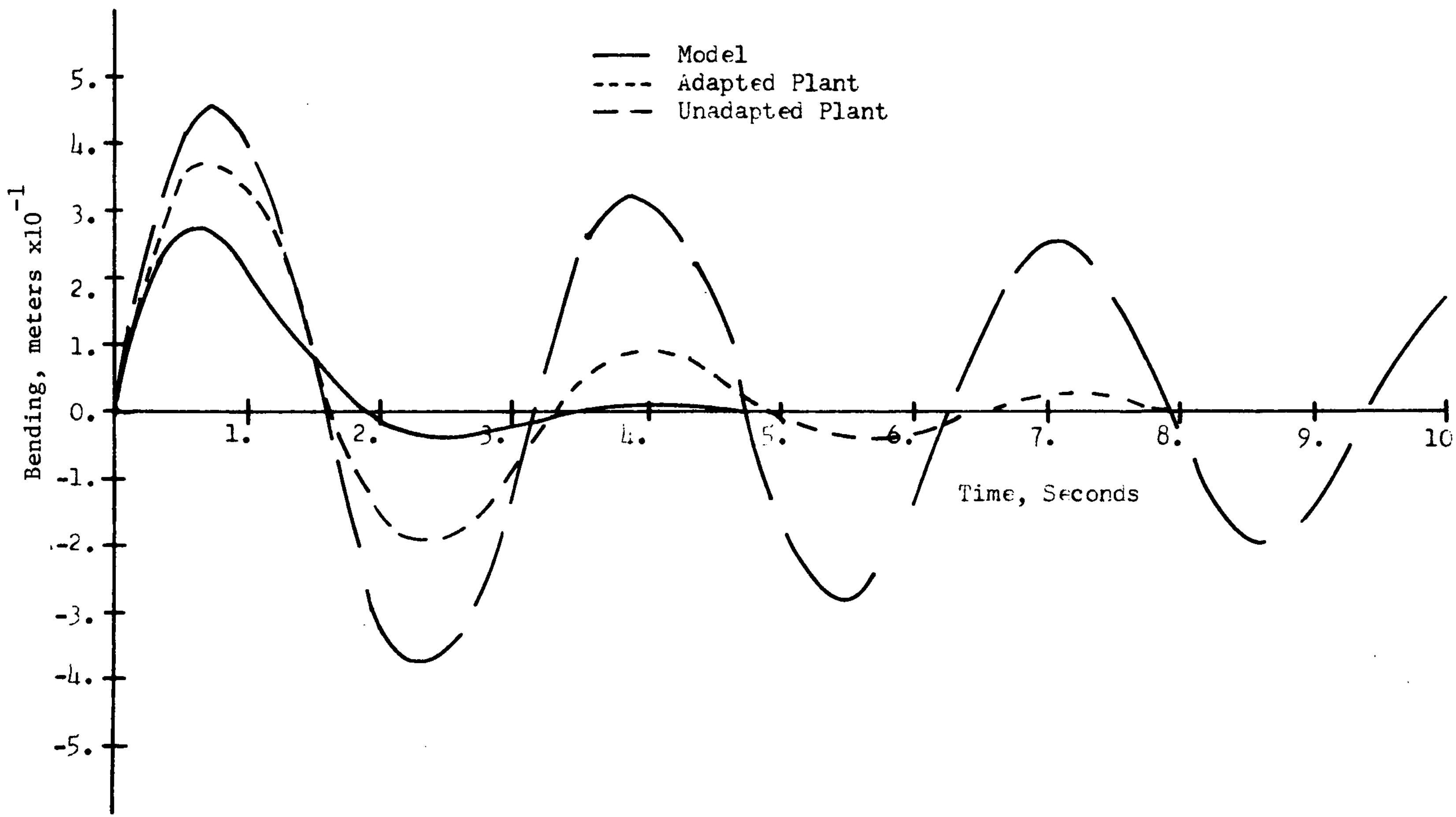


Figure III-2. **Bending mode responses of model, adapted plant, and unadapted plant, step input.**

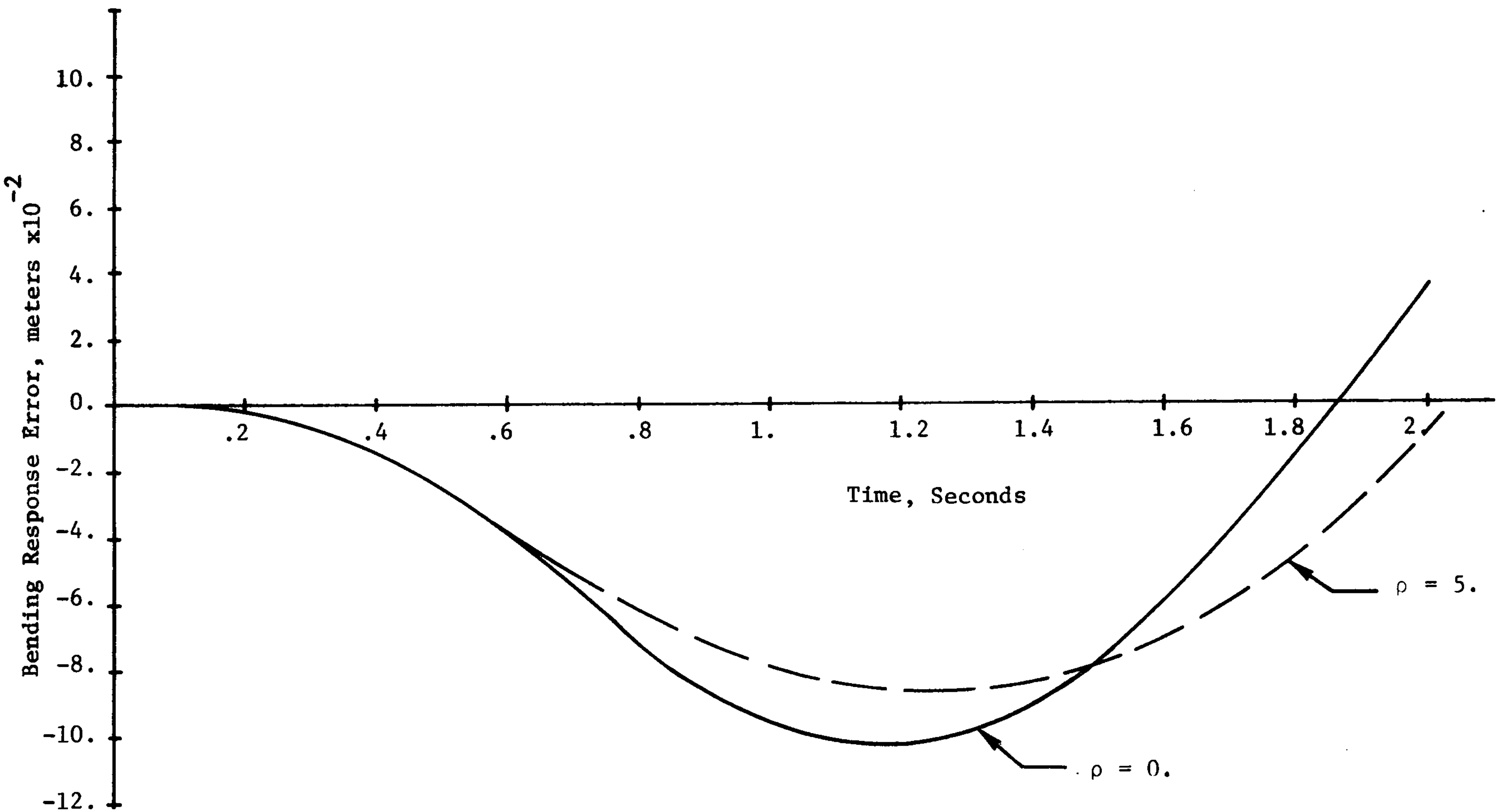


Figure III-3. Bending mode response error curves for sine wave input.

value of the response error. This is particularly significant because the smaller the response error the better the adaption is being accomplished which means that the plant is more accurately tracking the model.

Figures III-4 and III-5 show the rotational position and velocity responses, respectively, of the unadapted plant, the adapted plant and the model. For Figures III-4 and III-5, the input is a sine wave. For these figures, the adaptive gains are $\alpha_{33} = \alpha_{34} = \alpha_{43} = \alpha_{44} = 6.$, $\beta_{33} = \beta_{34} = \beta_{43} = \beta_{44} = 3.$, and $\rho_{33} = \rho_{34} = \rho_{43} = \rho_{44} = .2$. Figures III-6 and III-7 show the unadapted plant response, the adapted plant response, and the model rotational position and velocity responses for a step input to the system. The adaptive gains are the same as in Figures III-4 and III-5. As can be seen from these figures, the adaption is quite rapid and the plant tracks the model very accurately for either of the inputs. Figure III-8 shows the rotational position response error of the adapted plant for two values of the coefficient of the derivative term in Equations (III-18), (III-19), (III-20), and (III-21). As can be seen from the figure, the use of the derivative term significantly reduces the maximum value of the response error. The integral of the square of the bending response error and the rotational position response error were calculated using the values of ρ_{22}, ρ_{33}, ρ_{34}, ρ_{43}, and ρ_{44} given above. The results are given in Table 1. As can be seen from the table, using the derivative term of Equations (III-17), (III-18), (III-19), (III-20), and (III-21), also leads to a reduction in the integral of the square of the response error.

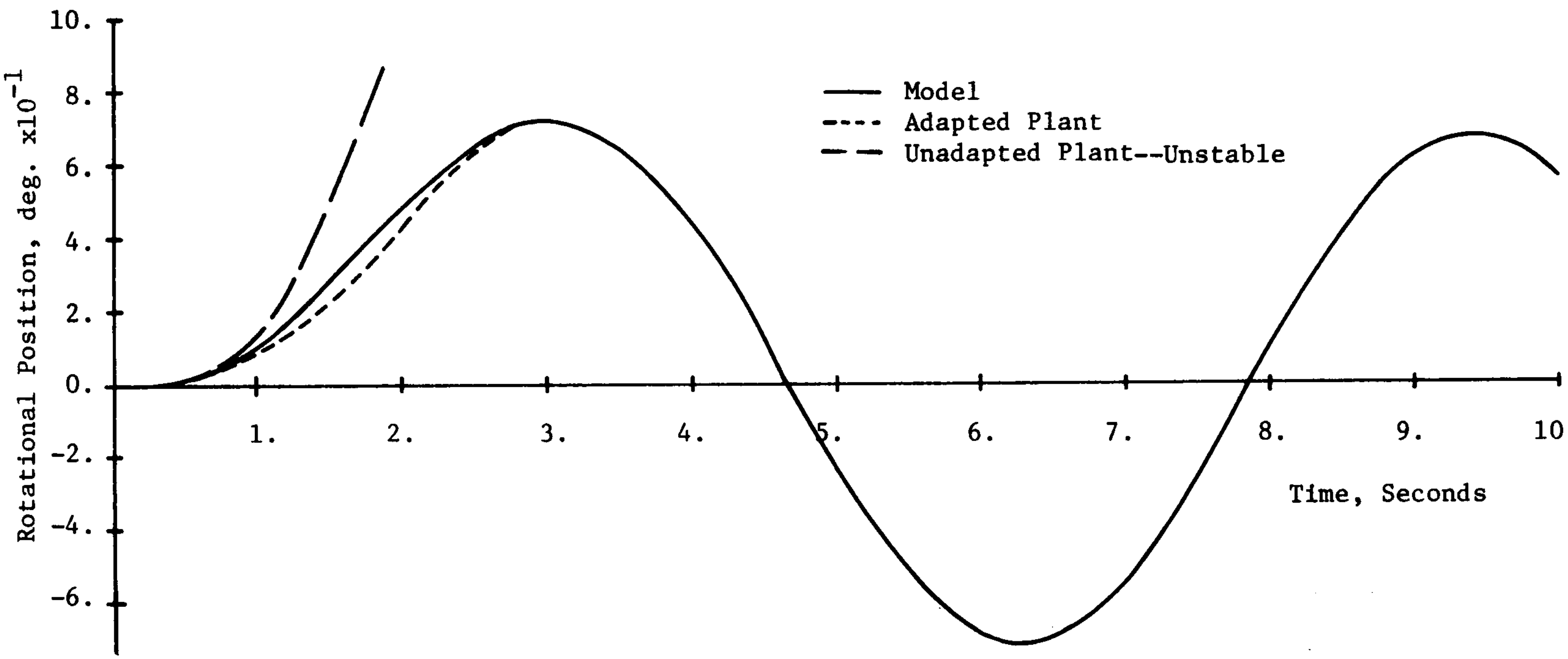


Figure III-4. Rotational position responses of model, adapted plant, and unadapted plant, sine wave input.

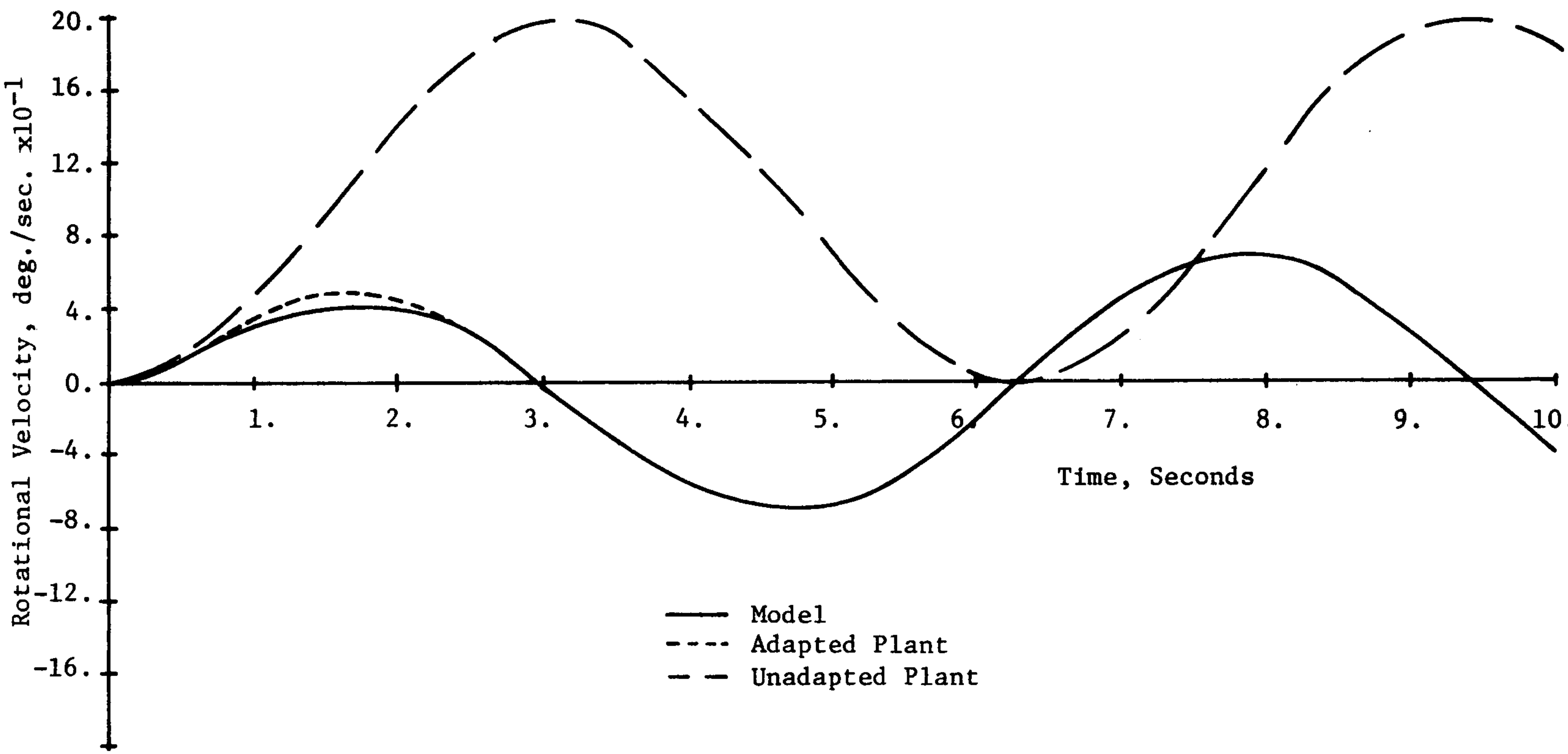


Figure III-5. Rotational velocity responses of model, adapted plant, and unadapted plant, sine wave input.

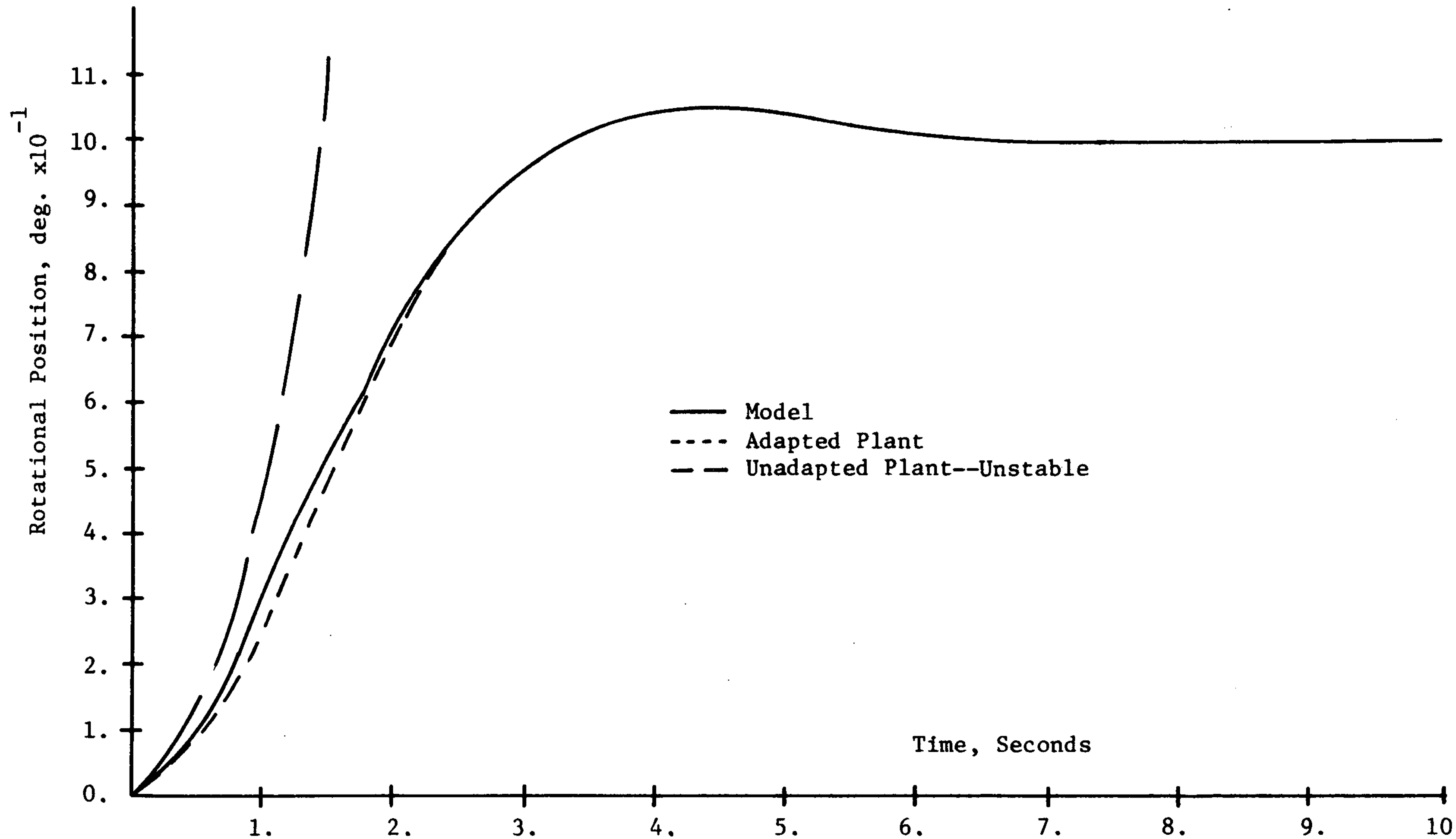


Figure III-6. Rotational position responses of model, adapted plant, and unadapted plant, step input.

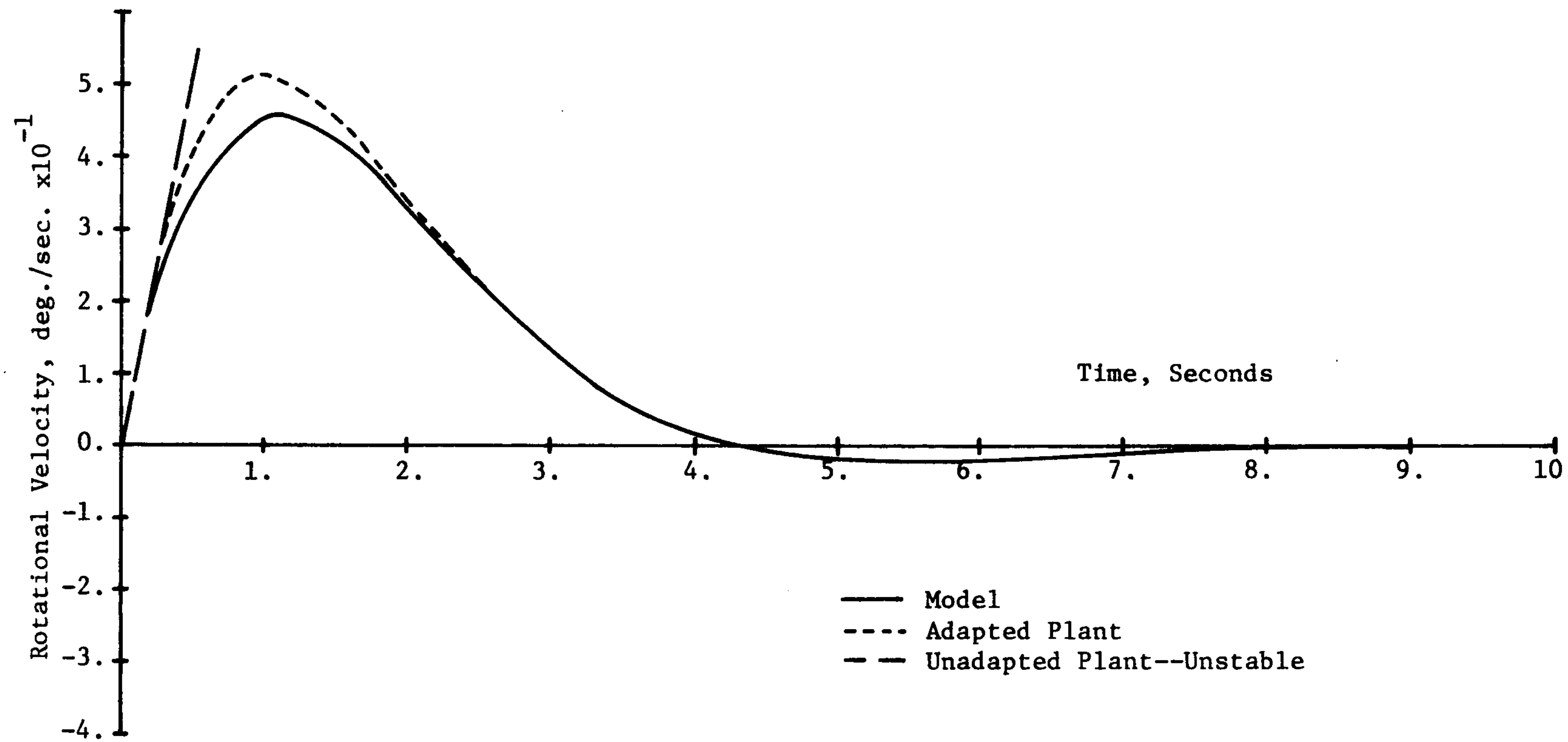


Figure III-7. Rotational velocity responses of model, adapted plant, and unadapted plant, step input.

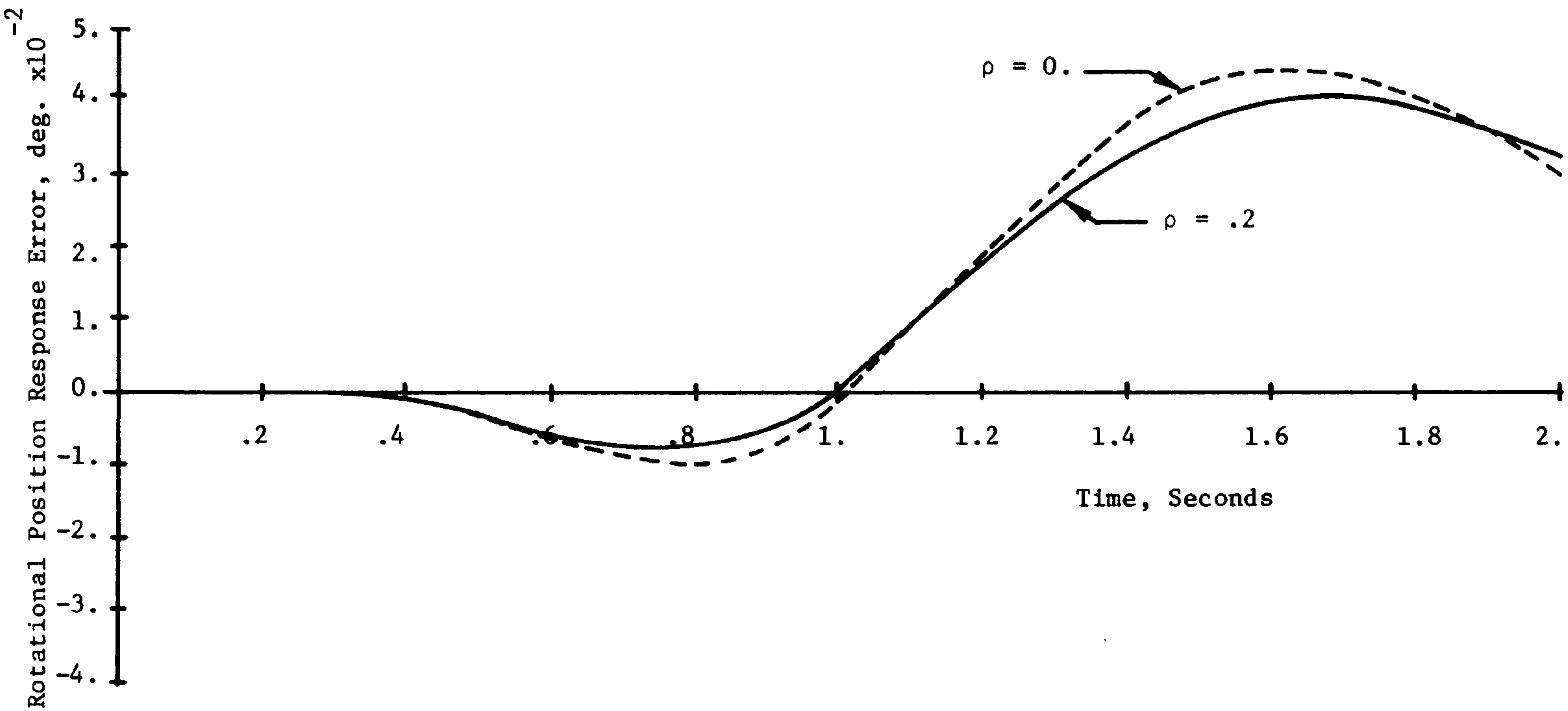


Figure III-8. Rotational position response error curves for sine wave input.

Table 1. Values of integral-square error for sine wave and step inputs.

$\int_0^{10} (\phi_m - \phi_p)^2 \, dt$ Sine wave input	$\int_0^{10} (\phi_m - \phi_p)^2 \, dt$ Step input	$\int_0^{10} (\eta_m - \eta_p)^2 \, dt$ Sine wave input	$\int_0^{10} (\eta_m - \eta_p)^2 \, dt$ Step input
$\rho = 0.$ 1.4313×10^{-3}	$\rho = 0.$ 1.1027×10^{-3}	$\rho = 0.$ 3.9880×10^{-2}	$\rho = 0.$ 3.9304×10^{-2}
$\rho = .2$ 1.3385×10^{-3}	$\rho = .2$ 1.0273×10^{-3}	$\rho = 5.$ 3.7779×10^{-2}	$\rho = 5.$ 3.8132×10^{-2}

The adaptive design technique developed in this chapter is applicable to plants with time-varying parameters. To illustrate this, simulation studies were made with the plant parameter a_{22} varying linearly from 0 to -.2 and with the plant parameter a_{34} varying linearly from 1. to .6 in 5. seconds. Figures III-9 and III-10 show the bending mode response of the unadapted plant, the adapted plant, and the model for a sine and a step input, respectively. As indicated by the figures, the adaption is rapid and the response error is very small. Figures III-11 and III-12 show the unadapted plant response, the adapted plant response, and the model response of the rotational position and rotational velocity for a sine wave input to the system and with the parameter a_{34} varying as noted above. As can be seen in both Figures III-11 and III-12, the response error is quite small and the adaption is rapid. Moreover, in Figure III-12, the scale is such that in order to show the unadapted plant response, the adapted plant and model response are virtually indistinguishable. Figure III-13 is thus included and shows the response error for the rotational velocity as a function of time. The figure shows that the adaption is rapid and that the plant tracks the model after a relatively short time. Figures III-14 and III-15 show the unadapted plant response, the adapted plant response, and the model response for the time-varying system with a step input. As in the case of the sine wave input to the system, the adaption is rapid and the plant tracks the model in a satisfactory manner.

In general, a control system may be considered as having two inputs: the first being a control or command input which may, in general, be

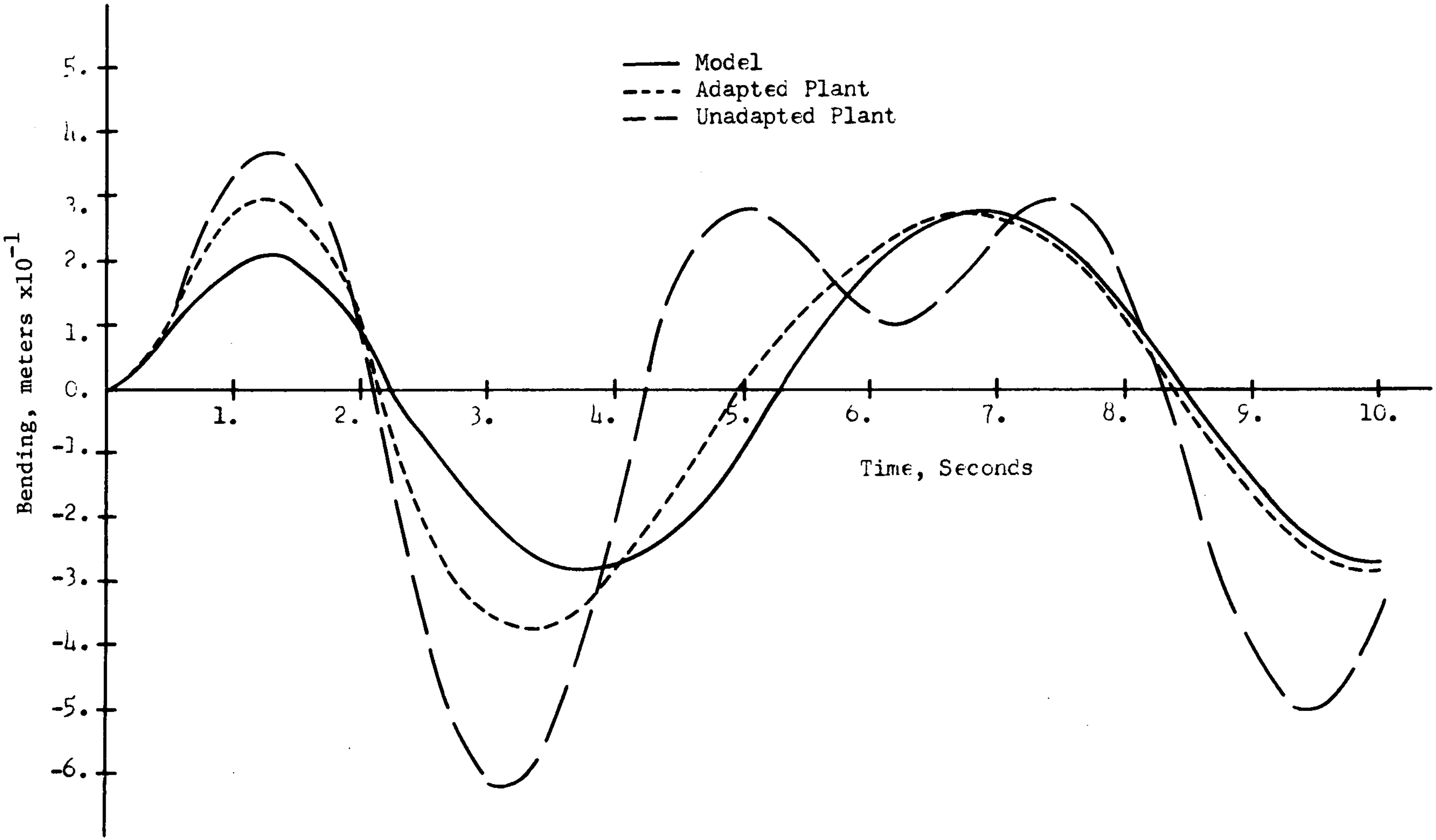


Figure III-9. Bending mode responses of model, adapted plant, and unadapted plant, sine wave input, time-varying parameters.

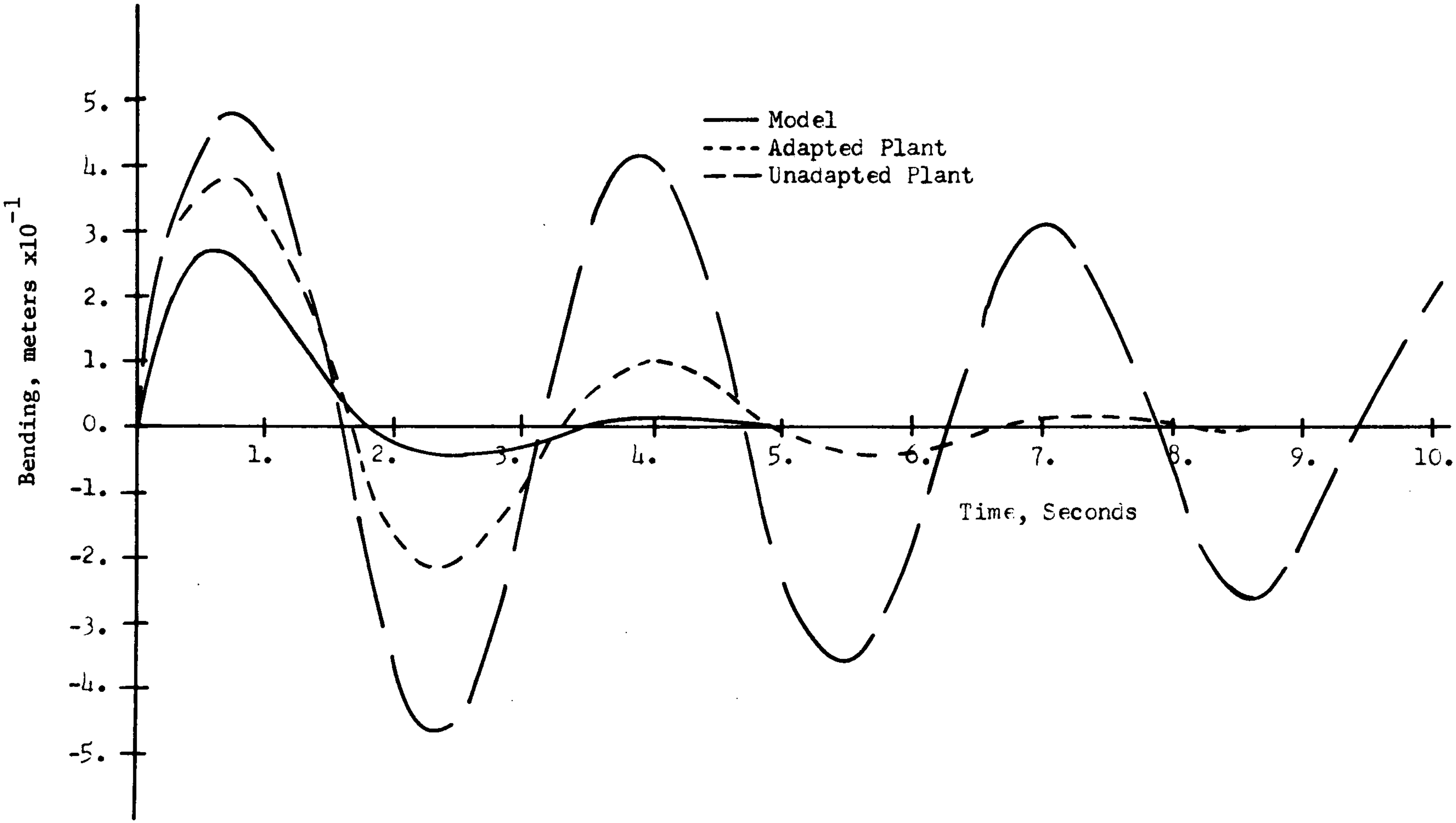


Figure III-10. Bending mode responses of model, adapted plant, and unadapted plant, step input, time-varying parameters.

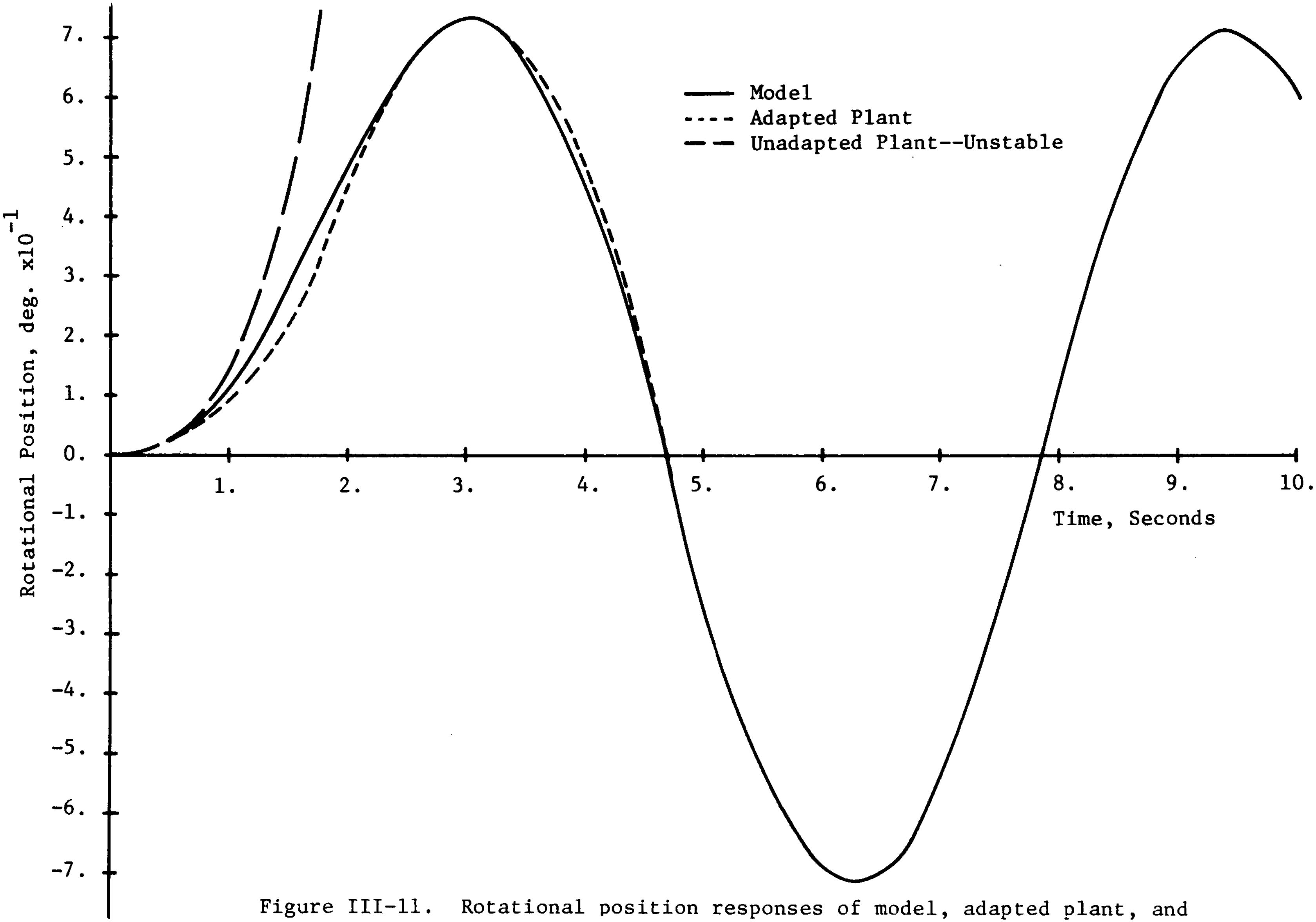


Figure III-11. Rotational position responses of model, adapted plant, and unadapted plant, sine wave input, time-varying parameters.

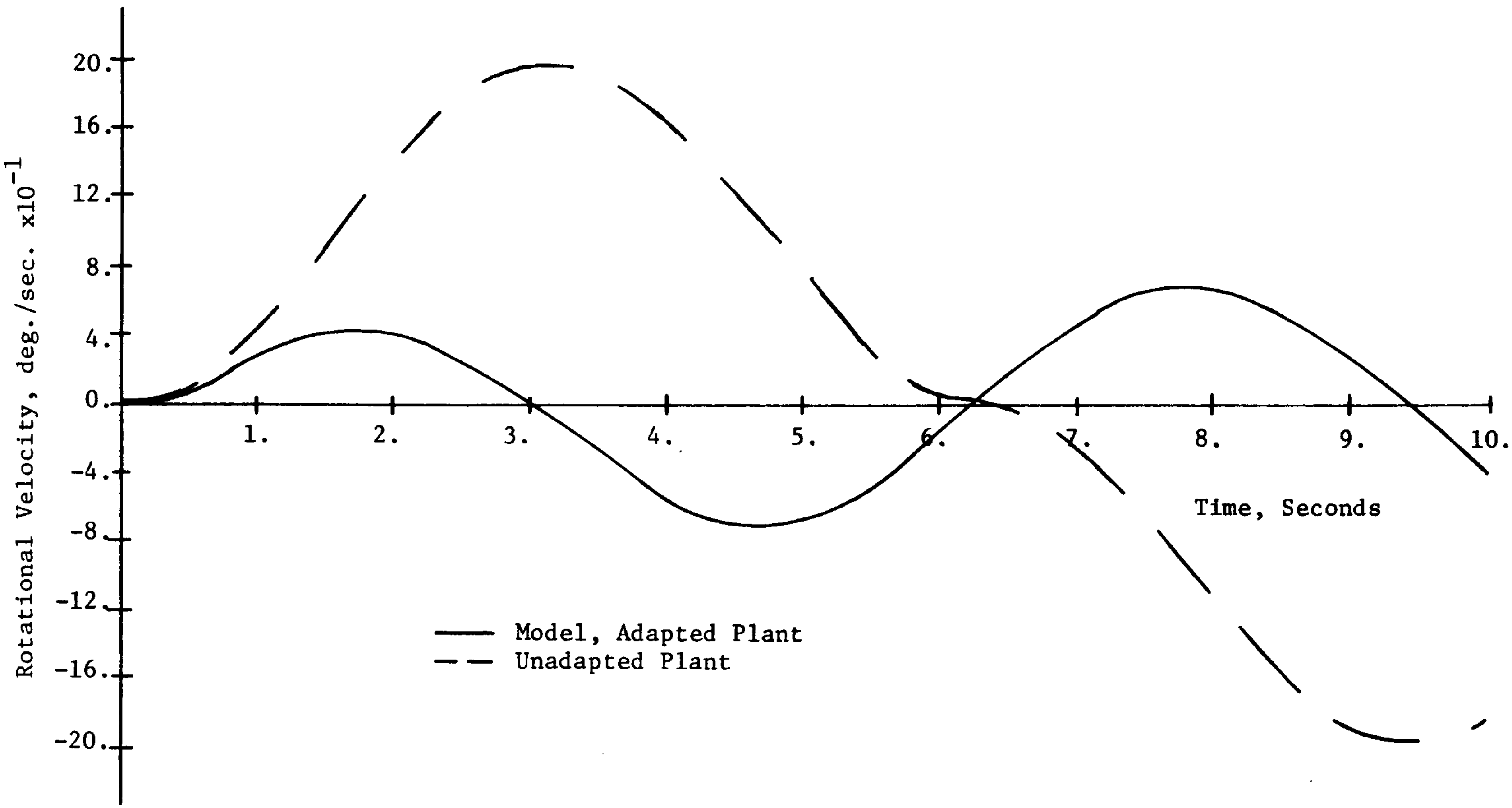


Figure III-12. Rotational velocity responses of model, adapted plant, and unadapted plant, sine wave input, time-varying parameters.

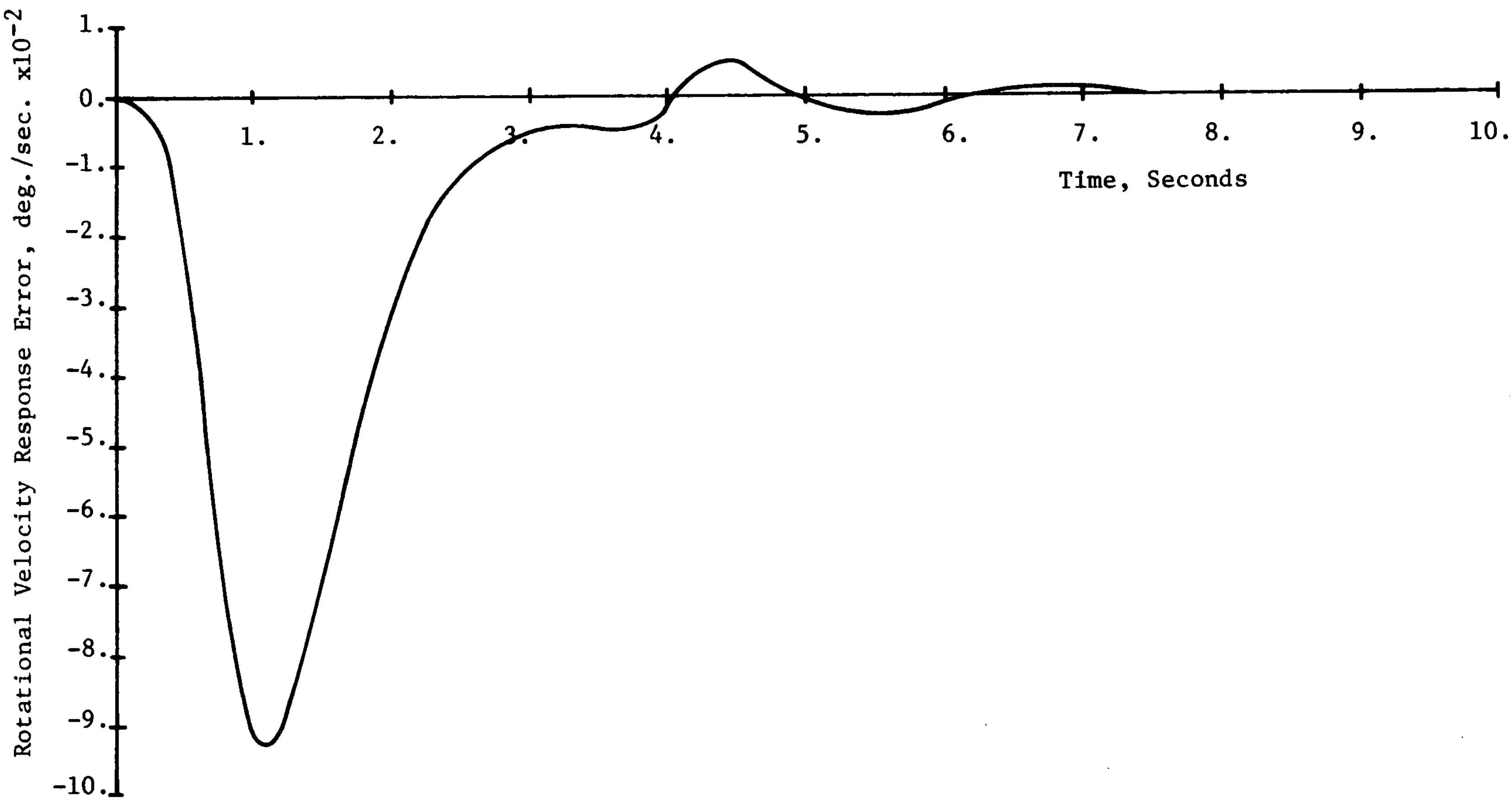


Figure III-13. Rotational velocity response error, sine wave input, time-varying parameters.

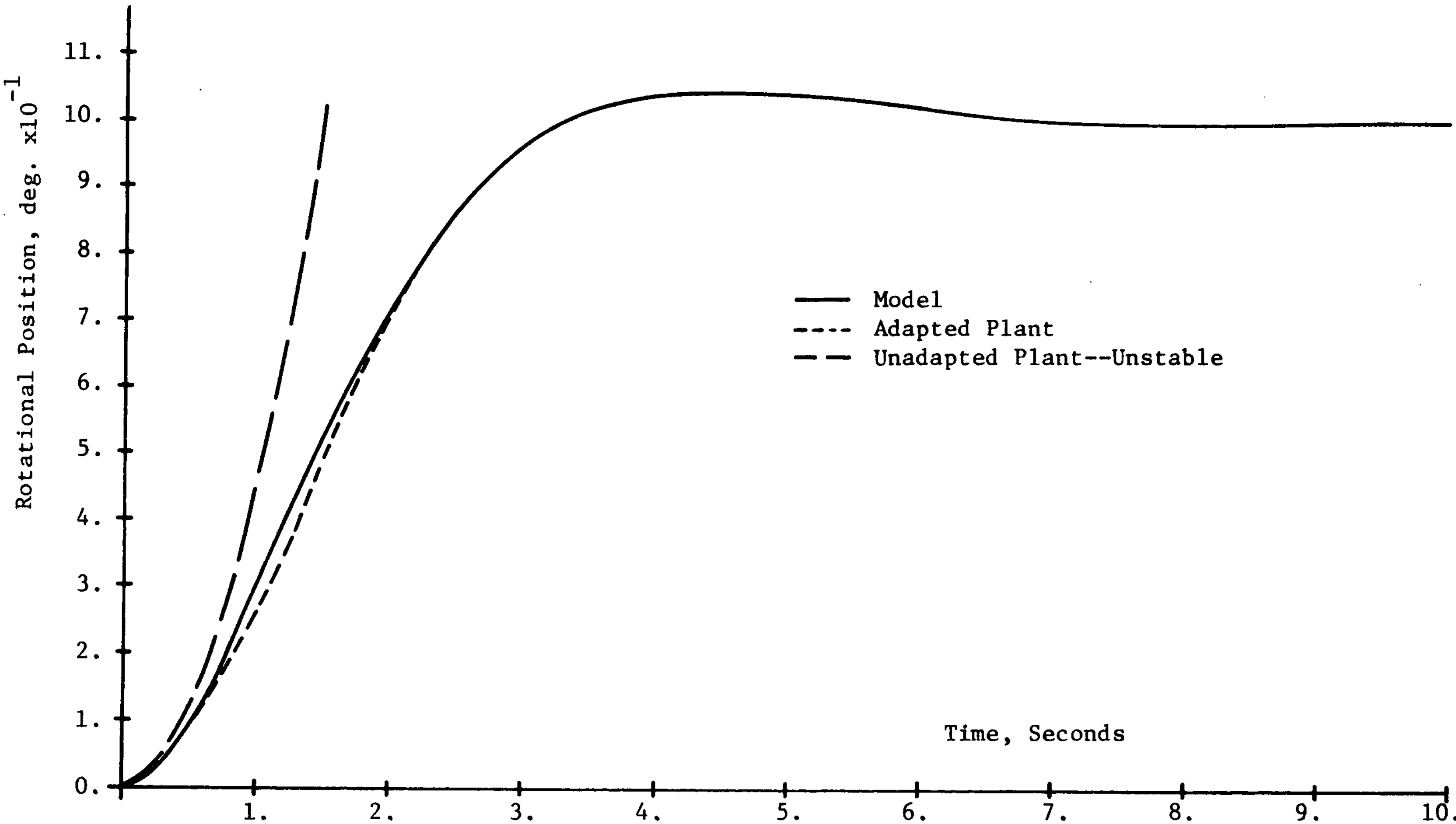


Figure III-14. Rotational position responses of model, adapted plant, and unadapted plant, step input, time-varying parameters.

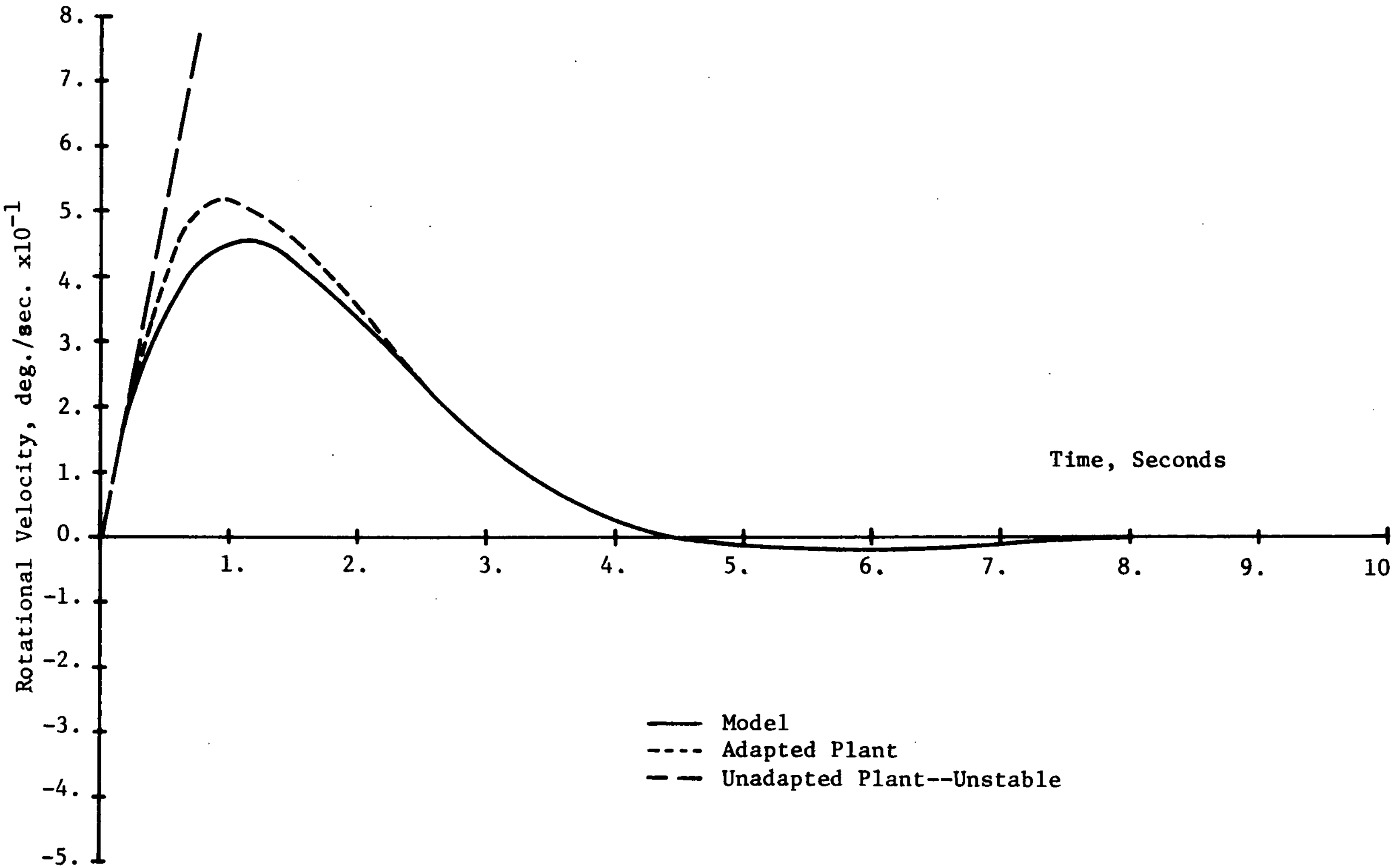


Figure III-15. Rotational velocity responses of model, adapted plant, and unadapted plant, step input, time-varying parameters.

changed in some desired manner and the second being a disturbance input which reflects the presence of an internal or external effect on the system which cannot be manipulated as desired. One common disturbance for a system such as the Space Shuttle is wind gust.

One of the inherent capabilities of an adaptive control system is the ability to "adapt" so as to be able to provide the desired response characteristics regardless of disturbances, unknown parameters, etc. A simulation study was performed in order to determine the effect due to wind gusts on the Space Shuttle vehicle compensated by adaptive control. The wind gusts were simulated by a step input at 3. seconds with a magnitude of 50% of the maximum control force available. Figures III-16 and III-17 show the bending mode response of the unadapted plant, the adapted plant, and the model. The responses are shown for a wind gust in the positive direction in Figure III-16 and in the negative direction in Figure III-17. As can be seen from the figures, the adaption is very good and the plant closely tracks the model even with large disturbances. Figures III-18 and III-19 show the rotational position responses for the unadapted plant, the adapted plant, and the model for a positive wind gust and a negative wind gust, respectively. Figures III-20 and III-21 show the rotational velocity responses for the unadapted plant, the adapted plant, and the model for a positive wind gust and a negative wind gust, respectively. Again, the adaption is seen to be very satisfactory and the adapted plant is very adequately compensating for the disturbances. Figure III-22 shows a plot of the rotational position response error for no disturbance, a positive wind gust and a negative wind gust.

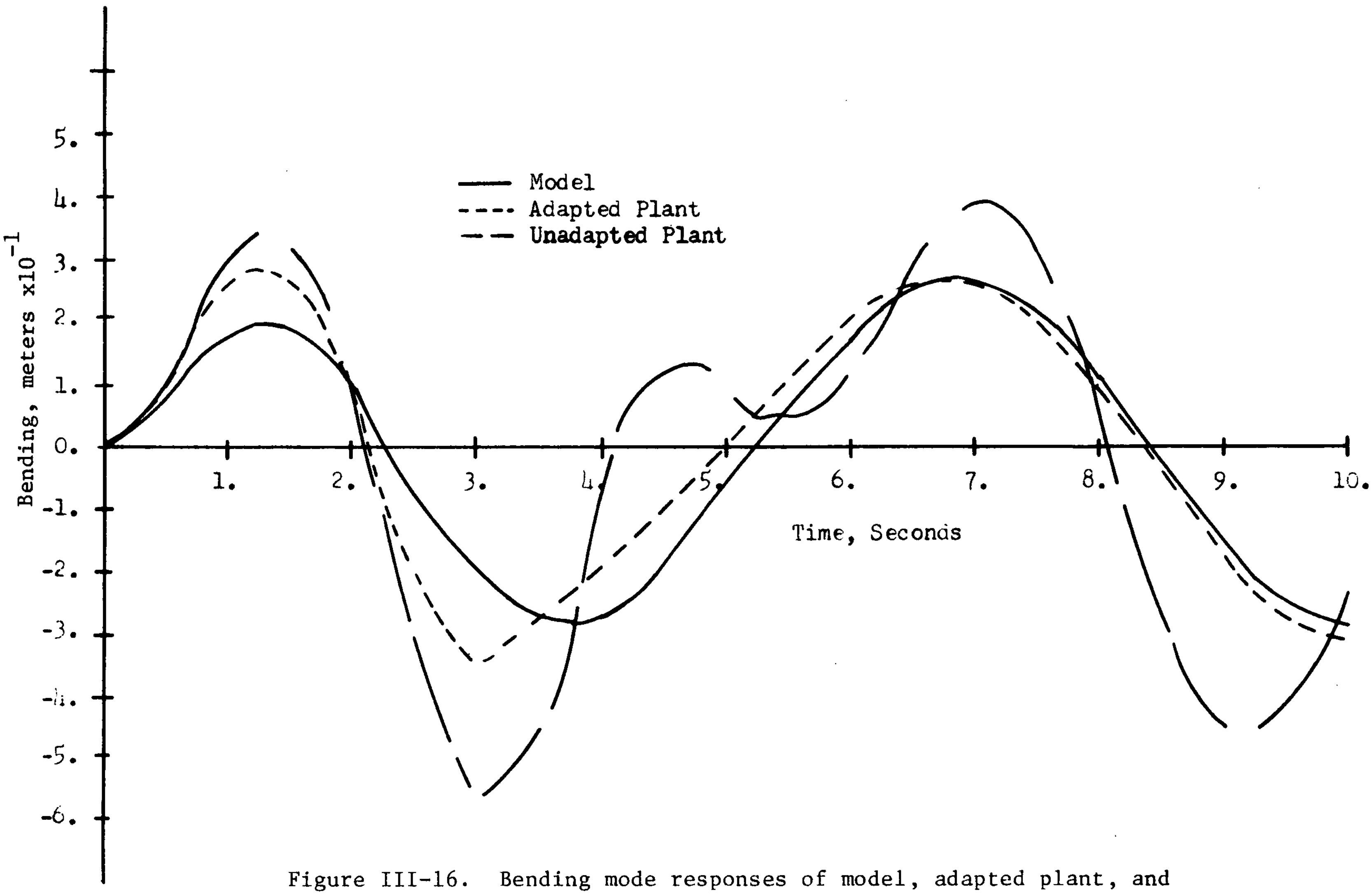


Figure III-16. Bending mode responses of model, adapted plant, and unadapted plant, positive wind gust.

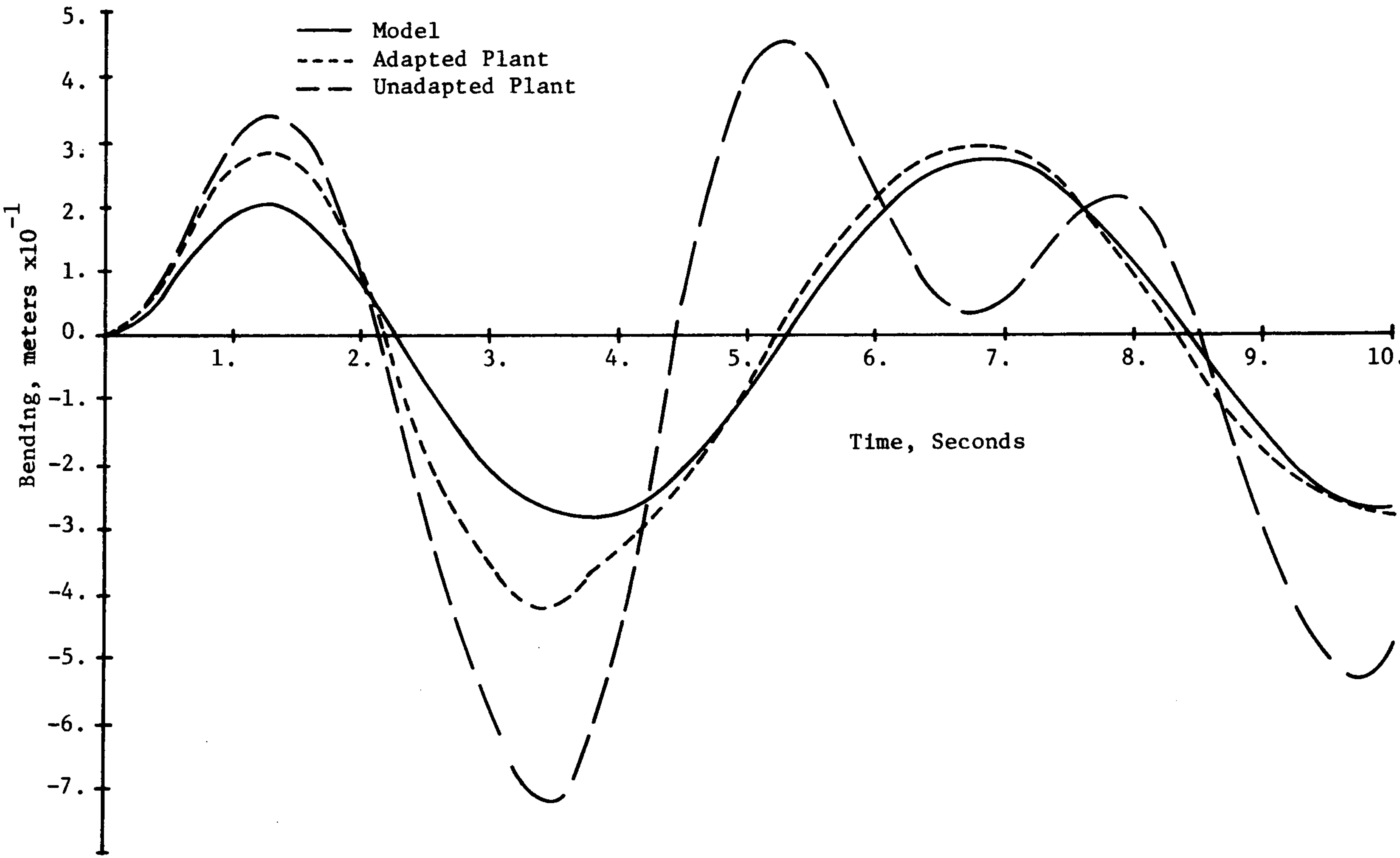


Figure III-17. Bending mode response of model, adapted plant, and unadapted plant, negative wind gust.

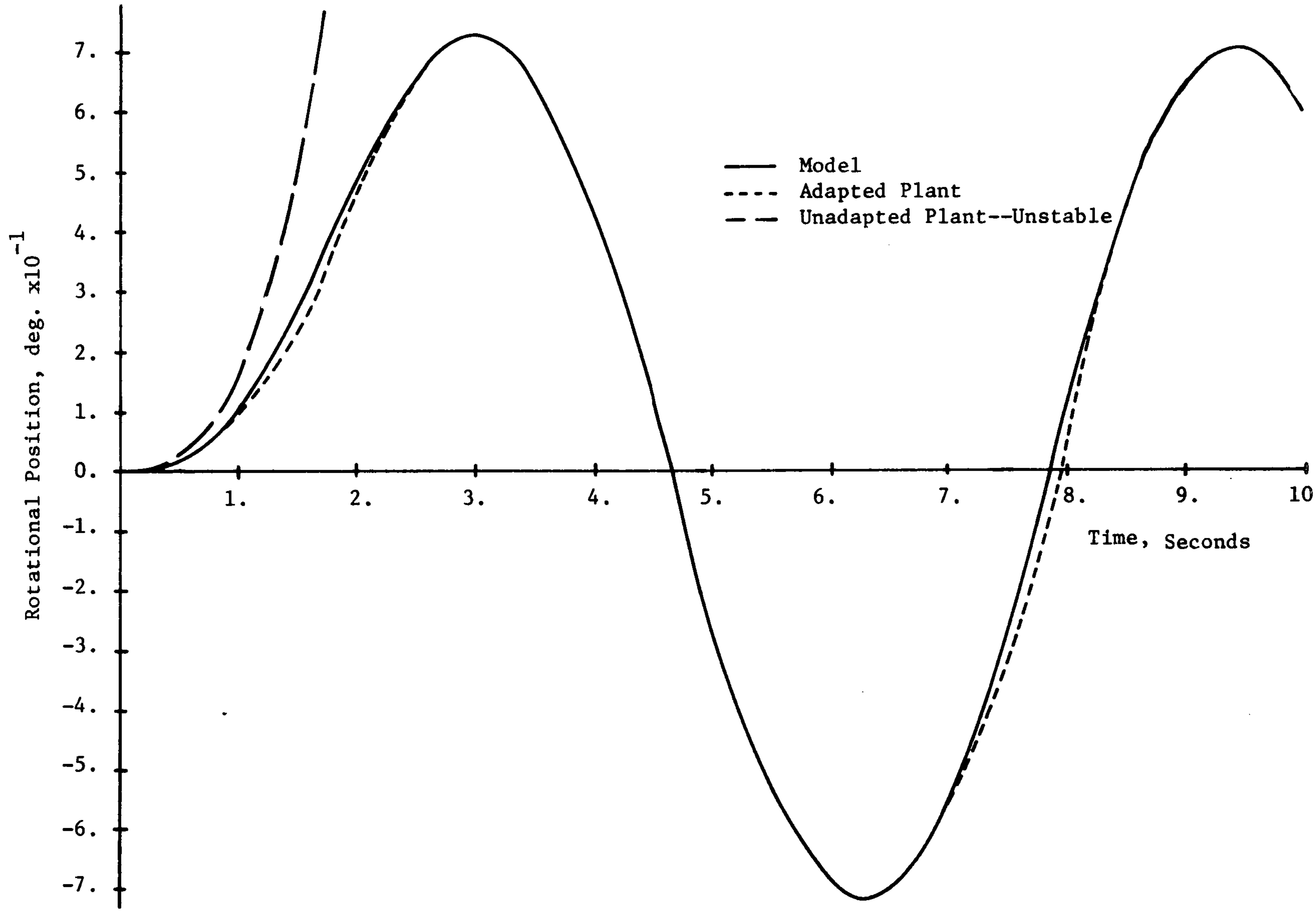


Figure III-18. Rotational position responses of model, adapted plant, and unadapted plant, positive wind gust.

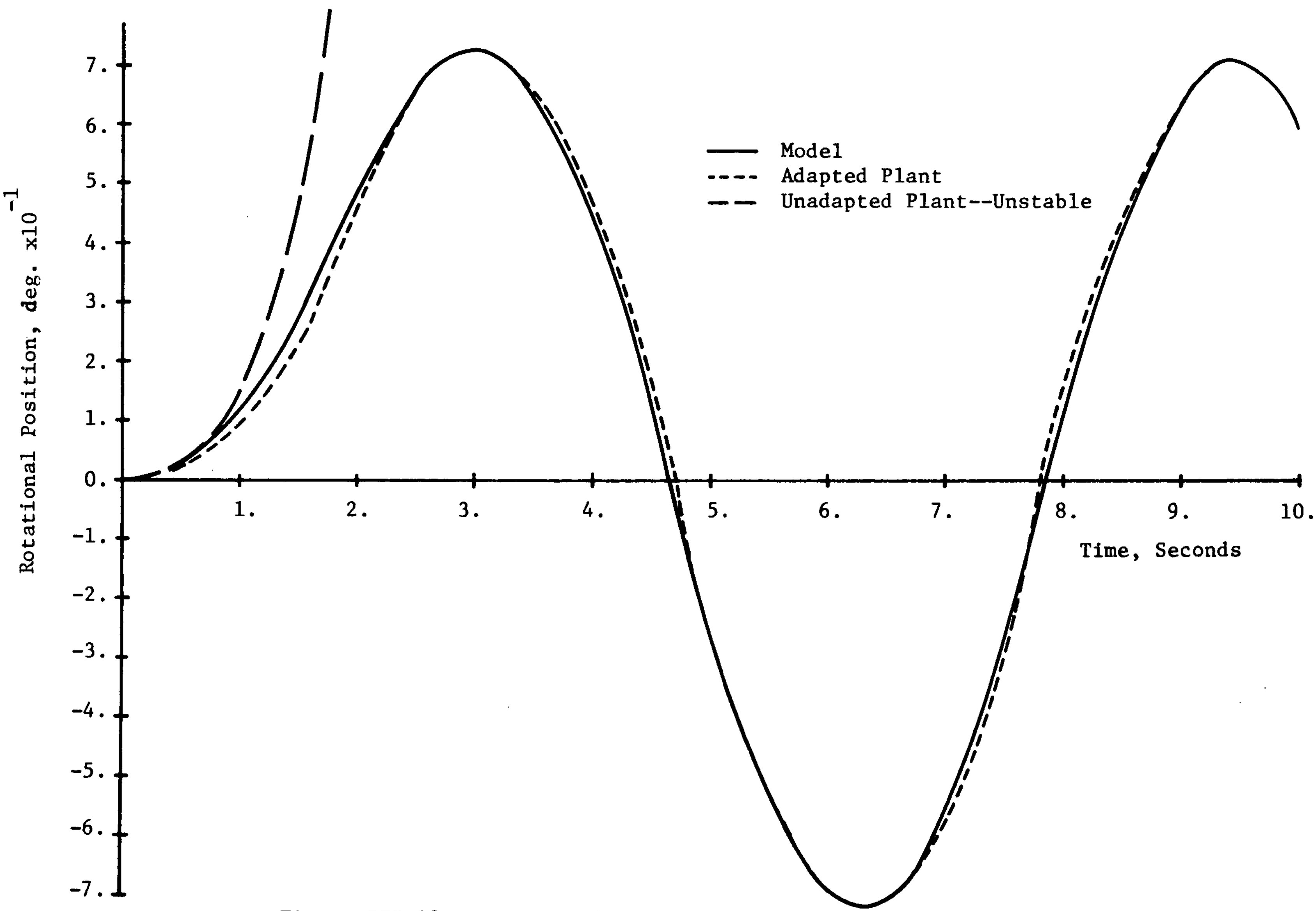


Figure III-19. Rotational position responses of model, adapted plant, and unadapted plant, negative wind gust.

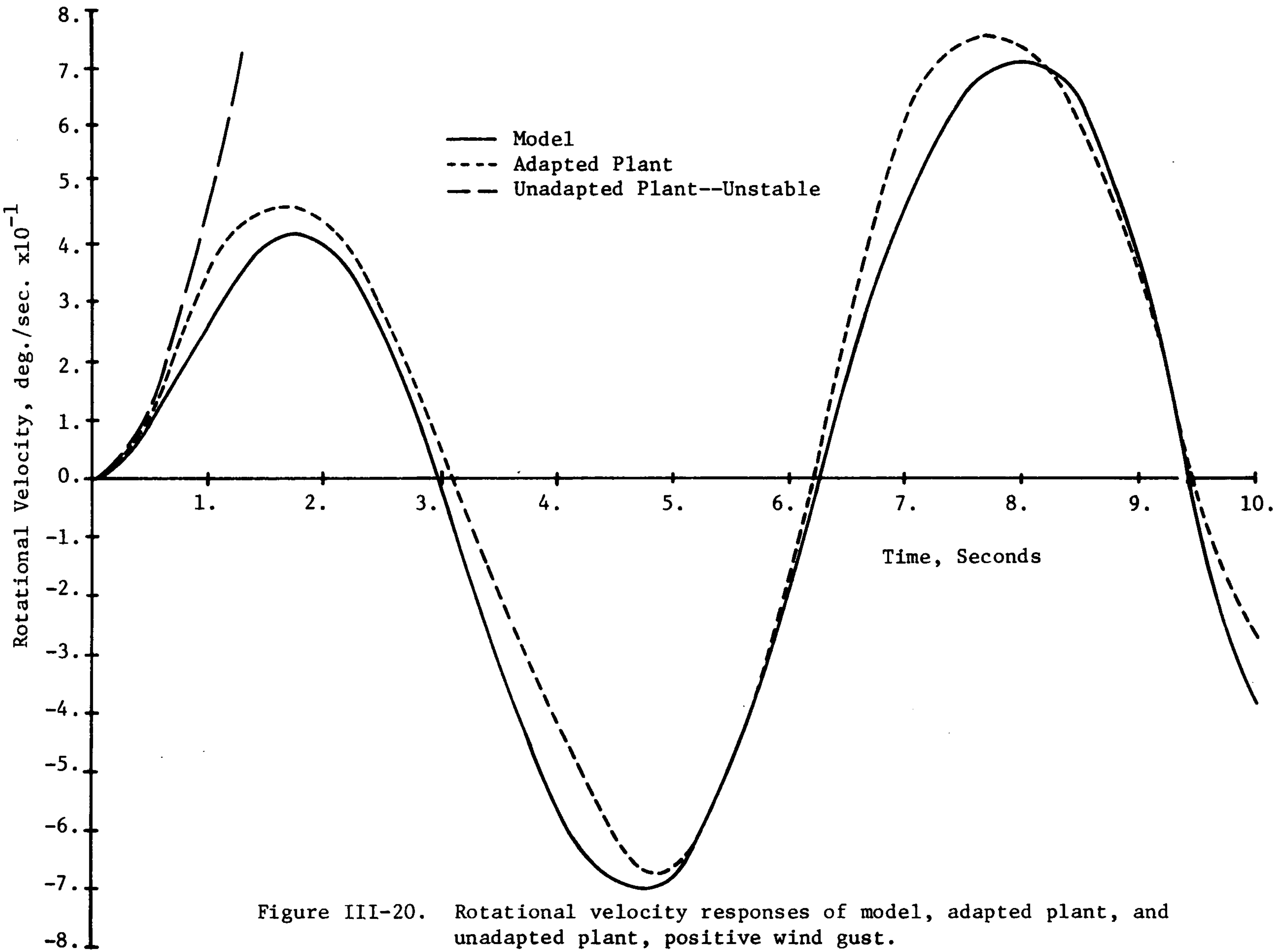


Figure III-20. Rotational velocity responses of model, adapted plant, and unadapted plant, positive wind gust.

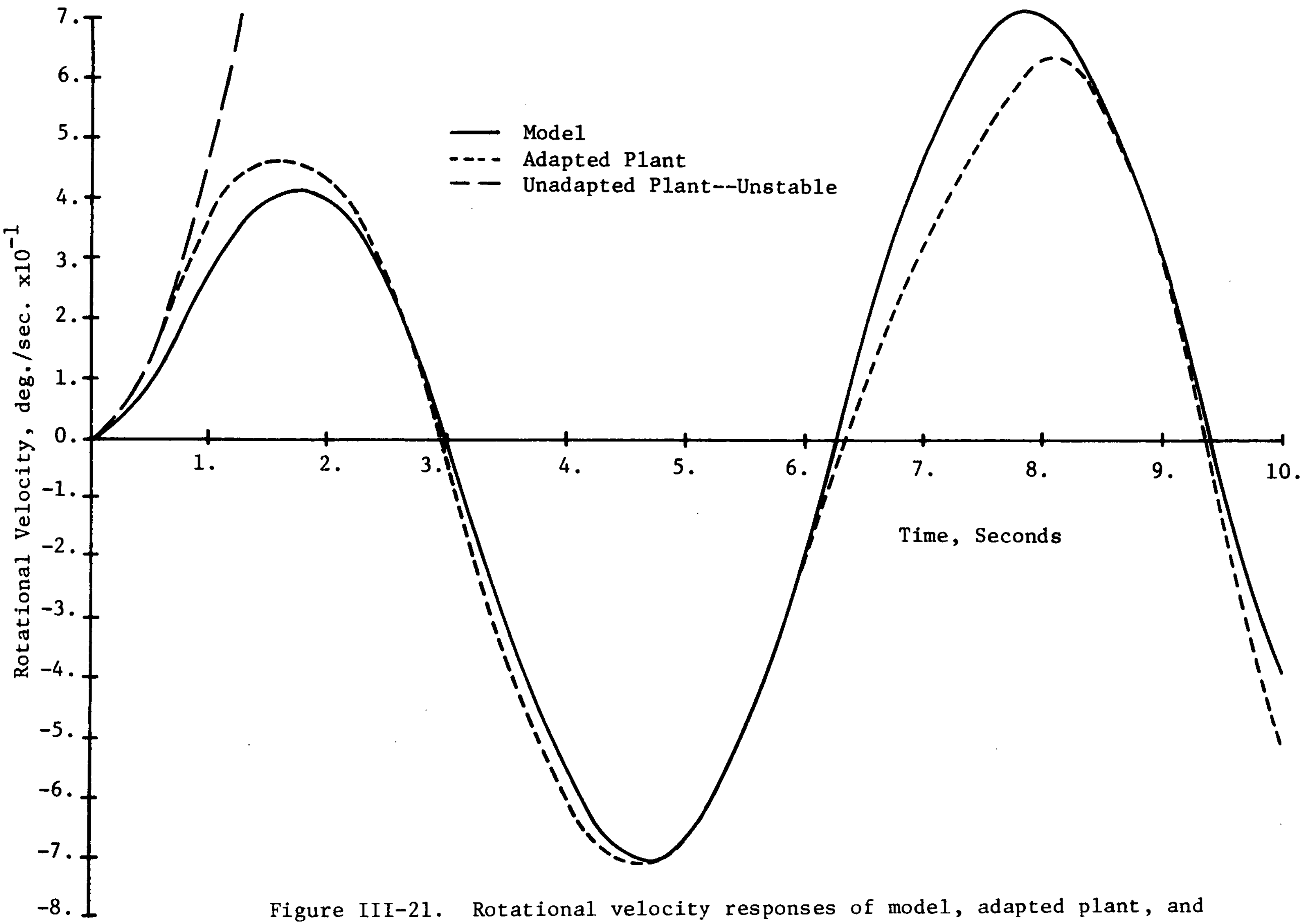


Figure III-21. Rotational velocity responses of model, adapted plant, and unadapted plant, negative wind gust.

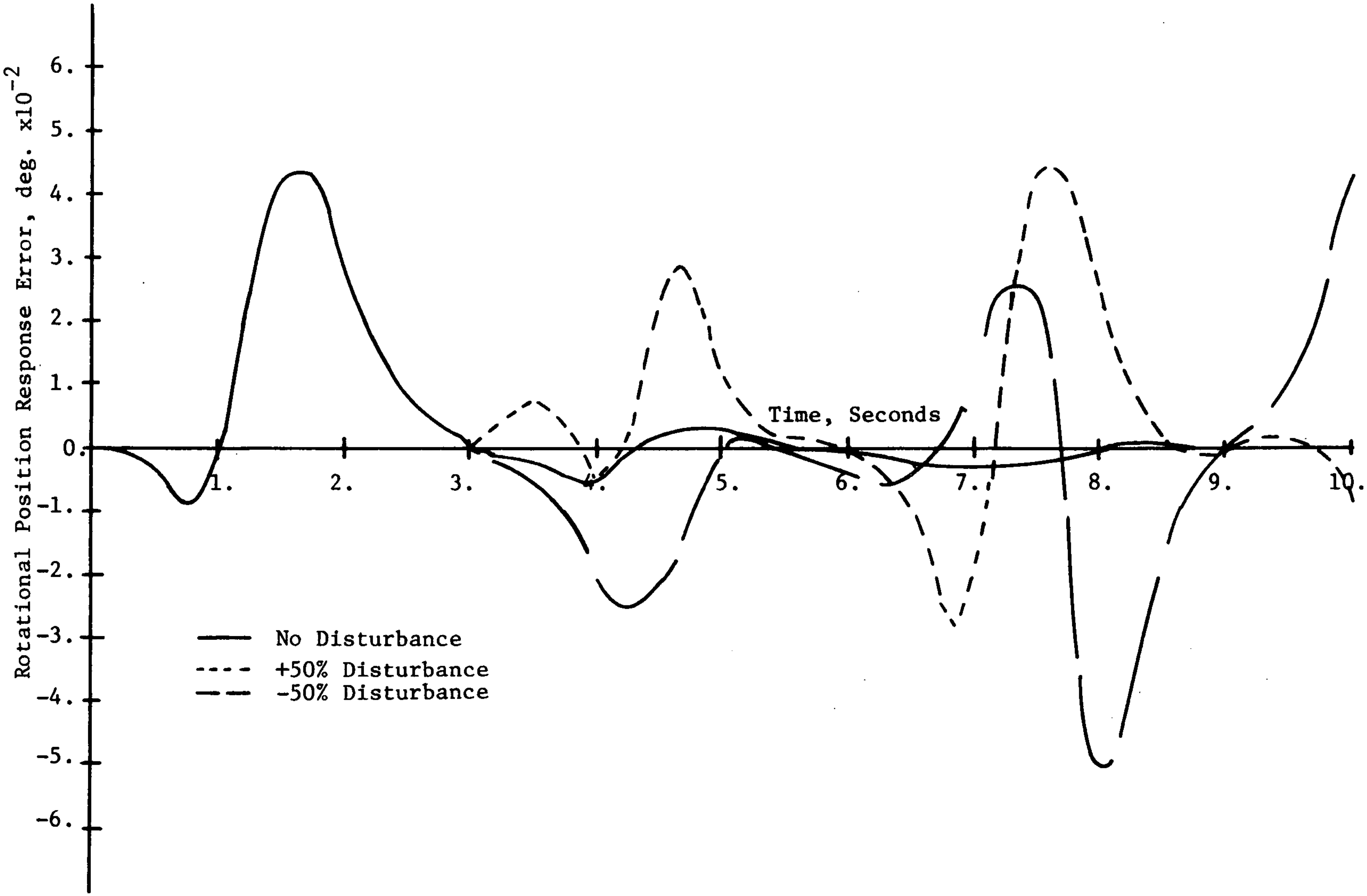


Figure III-22. Rotational position response error curves for no wind gust, positive wind gust, and negative wind gust.

The response error becomes small with the disturbances acting on the system as it does without them. Thus, the response error is quite small which means that the adaptive system is adequately compensating for the disturbance effects and is tracking the model.

A simulation study for the Space Shuttle vehicle was conducted with the thrust characteristics represented by a contactor mechanism with a dead-zone. The design technique developed in Section B of this chapter is applicable to this class of non-linear systems where the nonlinear and linear elements may be grouped as in Figure III-23.

The simulation was made using CSMP with the unadapted plant and model having the same dynamic characteristics as described above for the linear case. Figure III-24 shows the rotational position response versus time for the unadapted plant, the adapted plant, and the model with a step command to the system. As can be seen from the figure, the unadapted plant response is grossly different from the adapted response. The model and adapted plant rotational position responses are virtually equal. The rotational position response error is shown in Figure III-25 and can be seen to be quite small. Figure III-26 shows the phase-plant plot for the unadapted plant, the adapted plant, and the model. Figure III-26 shows that without adaptation, the system response is oscillatory whereas the adapted plant responds in a very satisfactory manner to the command input.

Figure III-27 shows the rotational position response for the unadapted plant, the adapted plant, and the model with an initial condition on the plant. The adapted plant is seen to have virtually the

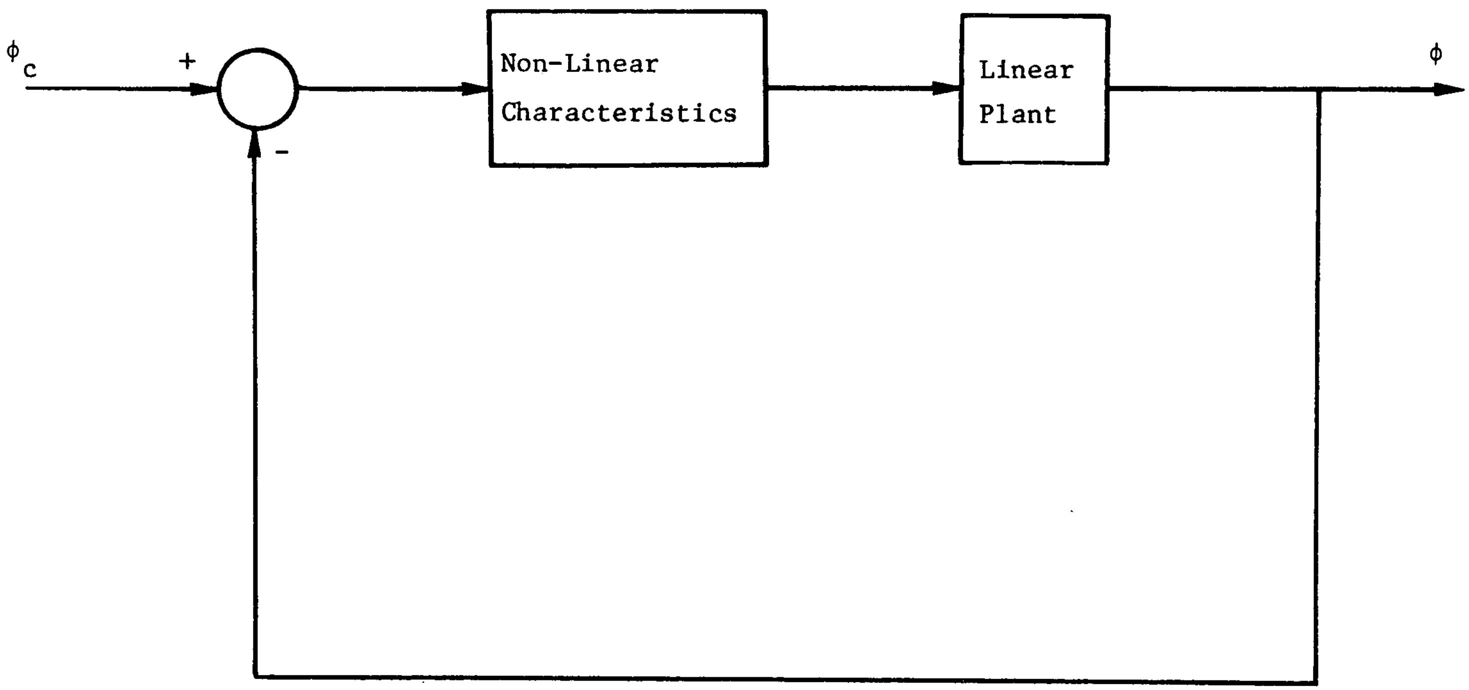


Figure III-23. Block diagram of non-linear system representation.

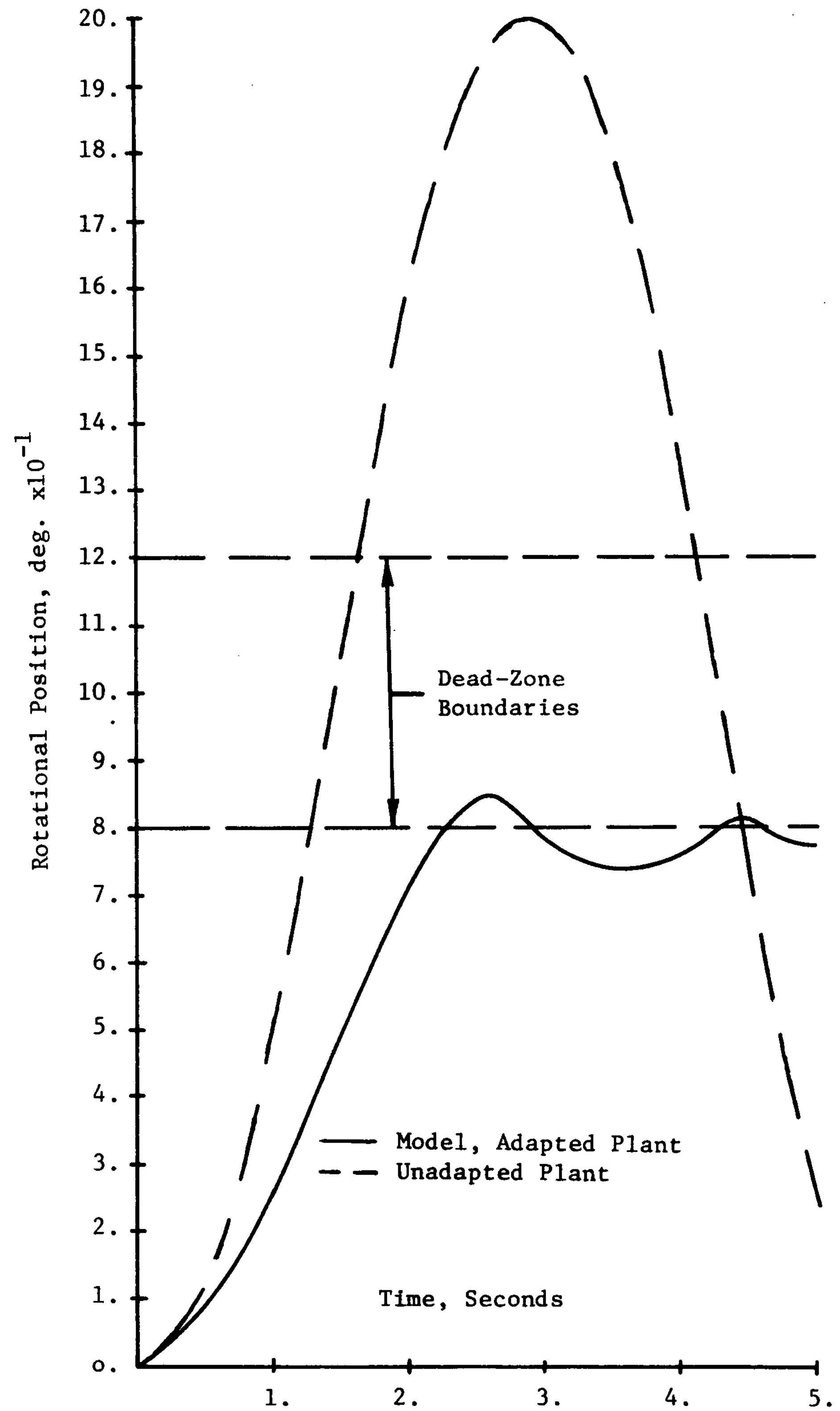


Figure III-24. Rotational position response curves of model, adapted plant, and unadapted plant, step command.

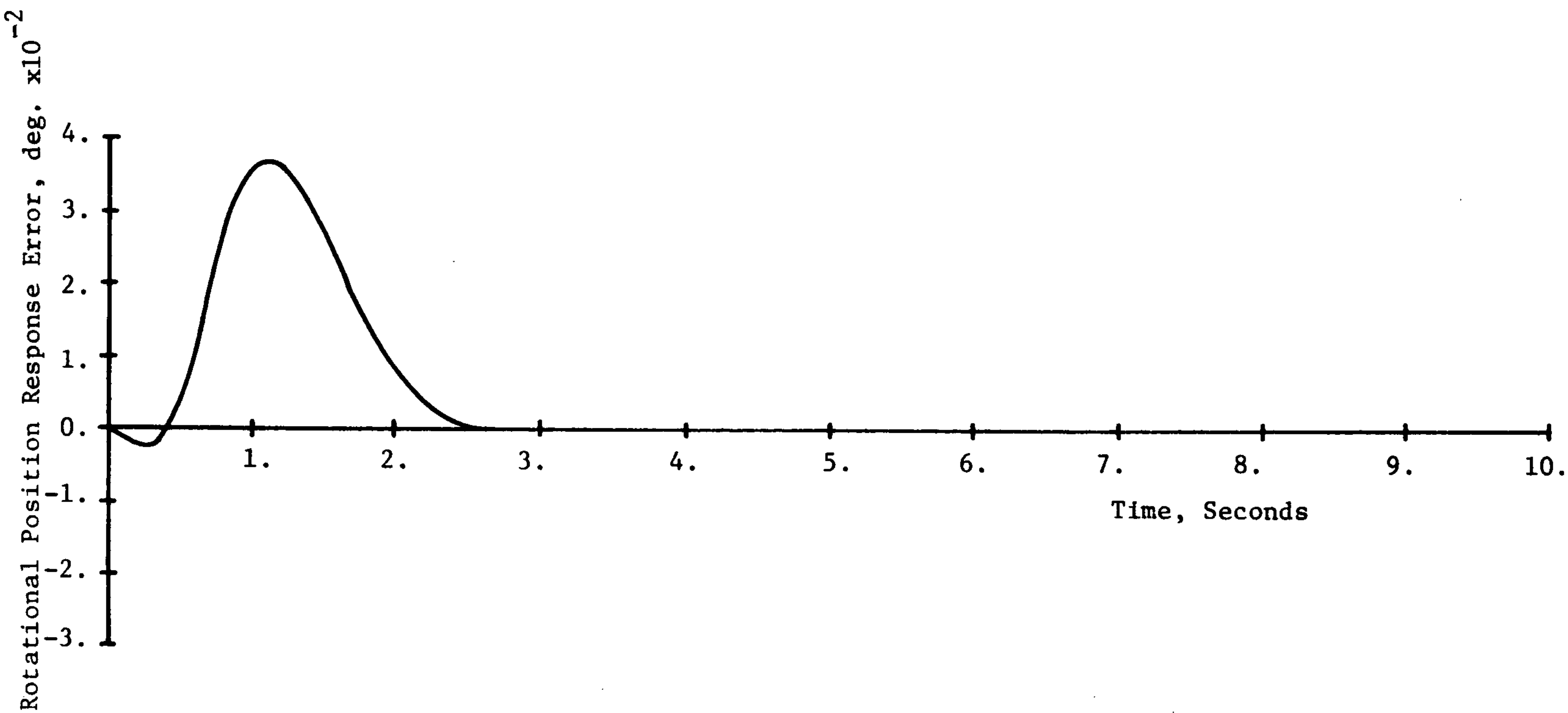


Figure III-25. Rotational position response error versus time, step command.

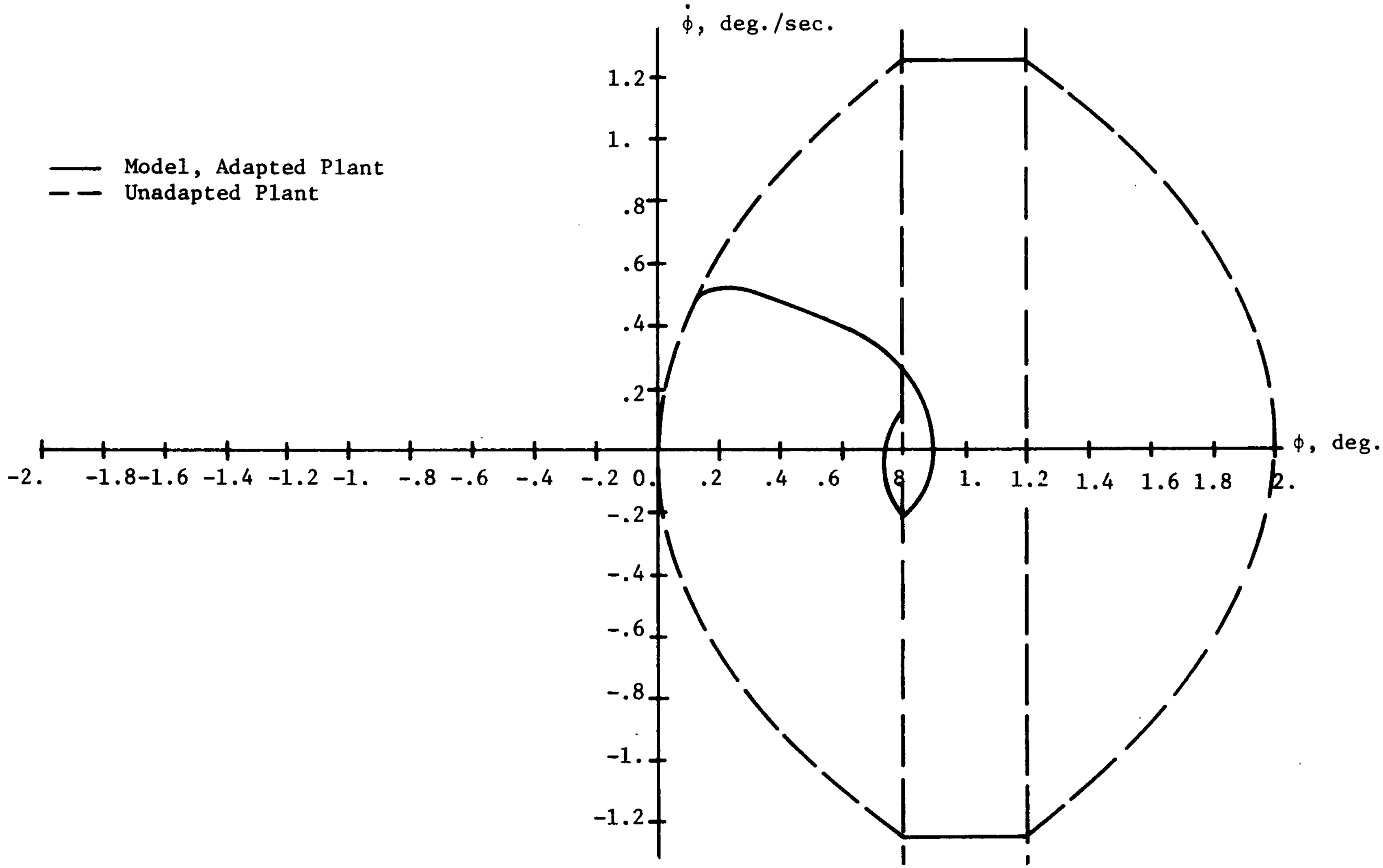


Figure III-26. Phase-plane plot of model, adapted plant, and unadapted plant, step command.

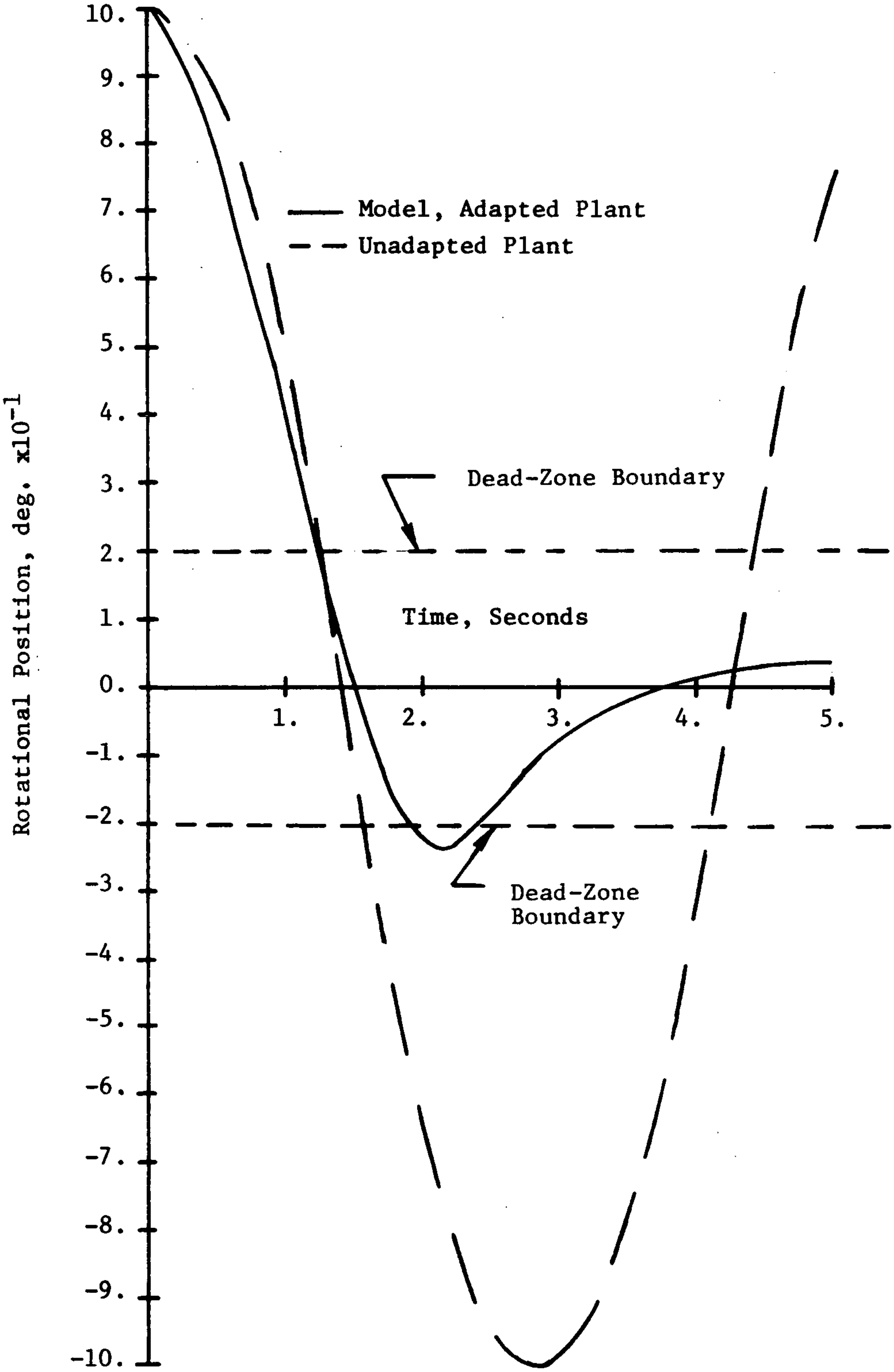


Figure III-27. Rotational position response curves of model, adapted plant, and unadapted plant, initial condition.

same characteristics as the model whereas the unadapted plant response is oscillatory. Figure III-28 shows the rotational position response error as a function of time. This shows that the plant is tracking the model very closely. Figure III-29 is a phase-plant plot for the adapted plant, the unadapted plant, and the model. As can be seen from the figure, the adapted system has good damping whereas the unadapted plant is highly oscillatory.

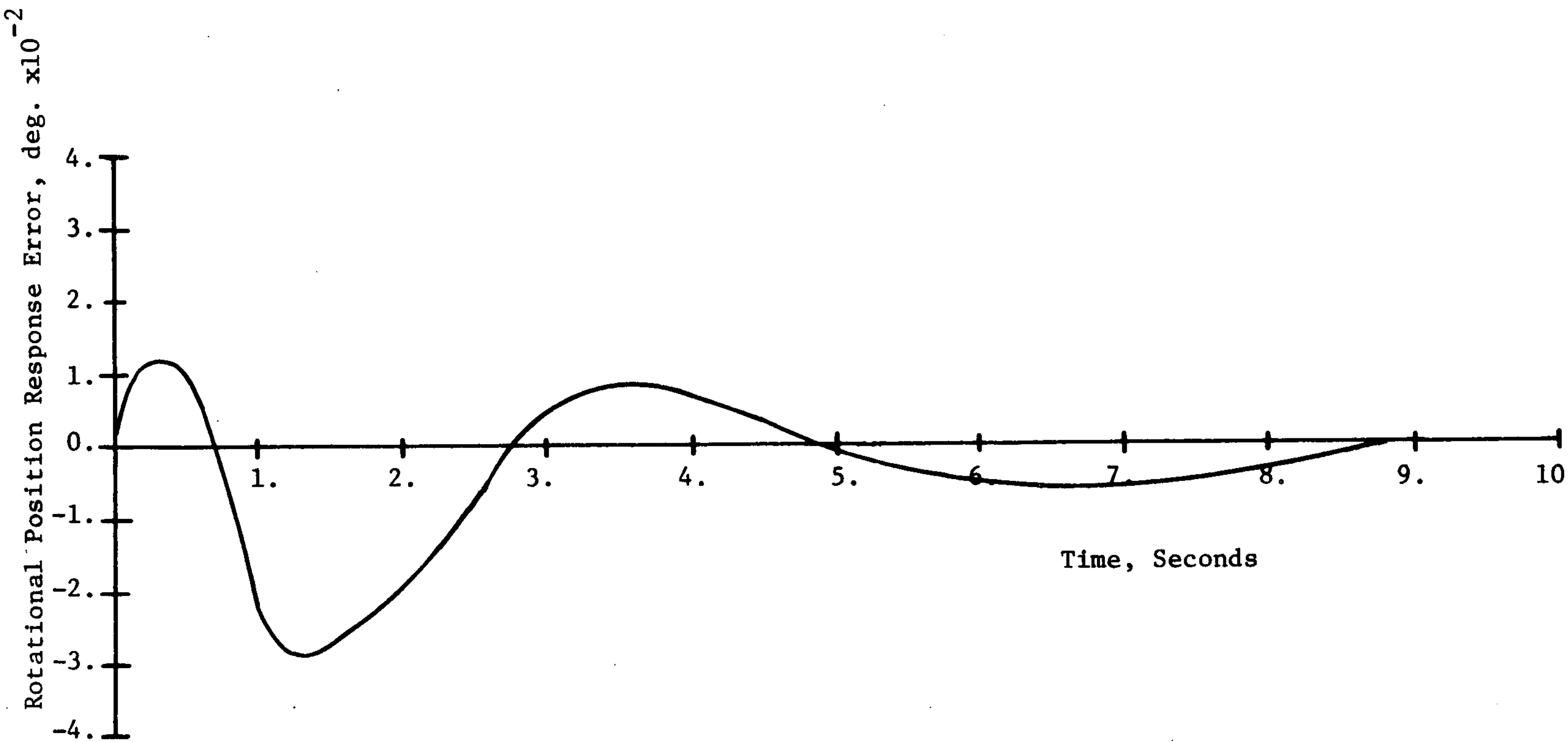


Figure III-28. Rotational position response error versus time, initial condition.

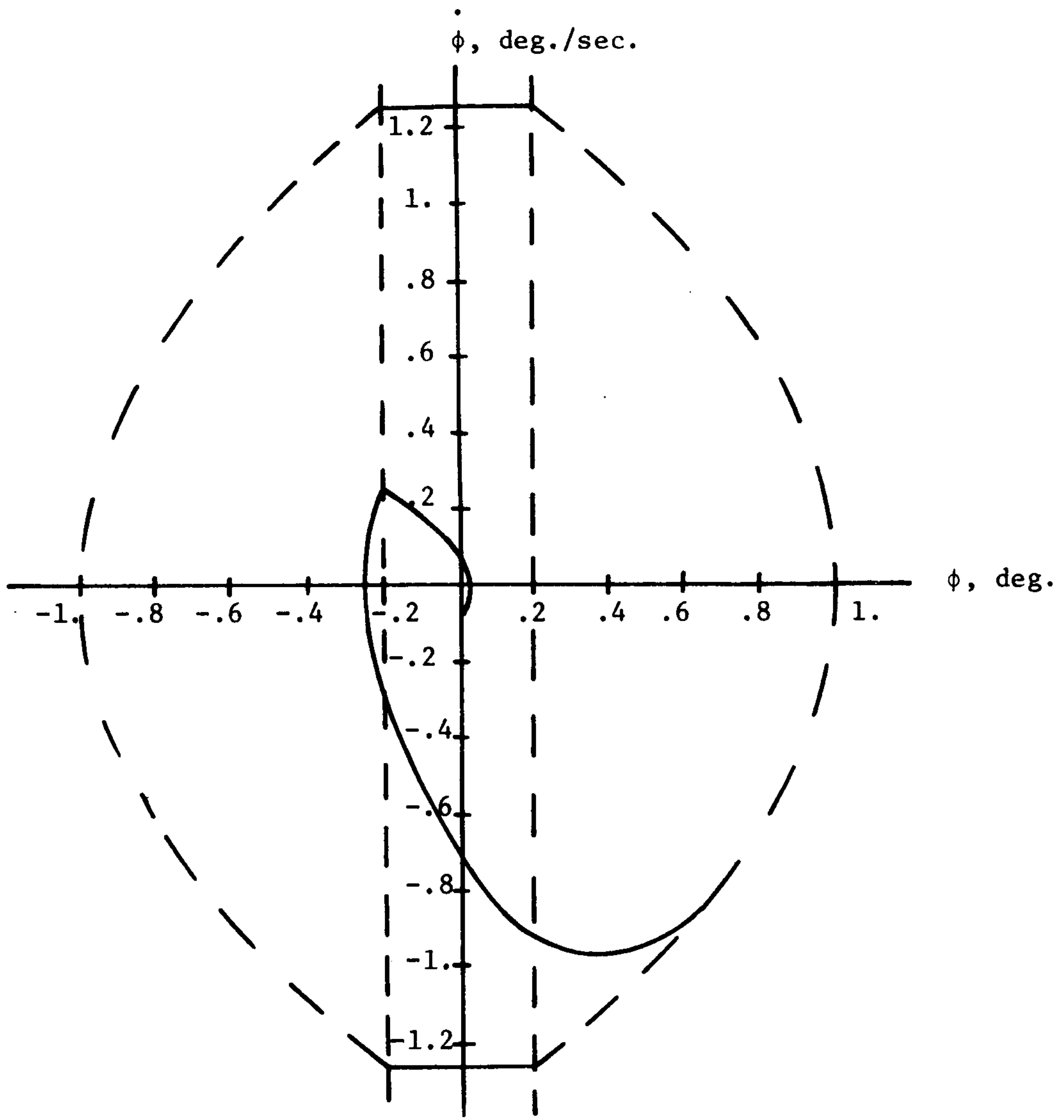


Figure III-29. Phase-plane plot of model, adapted plant, and unadapted plant, initial condition.

IV. FREQUENCY-DOMAIN MODEL REFERENCE ADAPTIVE CONTROL SYSTEM DESIGN

This chapter presents the results of the present research in the area of frequency-domain design of model-reference adaptive control systems. In some adaptive control system applications, it may be more desirable, from an implementation standpoint, to provide adaptation by some means other than the technique presented in Chapter III. The model-reference adaptive control system design technique presented in this chapter provides adaptation by means of pre-filters and feedback filters. This design technique is for systems represented by a linear, time-invariant plant as in [6]. Plant adaptation is accomplished by having feedback filters and pre-filters with filter gains which are determined by the application of Liapunov's direct method to the system error equations. As a result of the application of Liapunov's direct method, the response error (model output minus plant output) is asymptotically stable. The filters are designed so that the denominator of each filter transfer function is the same as the denominator of the model transfer function. This enables the basic filter networks to be utilized for various missions provided that the system model is valid for the different missions. The design technique is applied, as was the design method presented in Chapter III, to the Space Shuttle vehicle with linear characteristics and to the Space Shuttle vehicle with non-linear characteristics. The effect of wind gust disturbances on the vehicle was also investigated as was the case of time-varying parameters. Simulation results of the Space Shuttle vehicle show that the adaption is quite rapid and that the response error of the adapted system is greatly reduced as compared to the response error of the unadapted system.

A. Problem Formulation

The plant is assumed to be a linear time-invariant system which may be represented in the frequency-domain as:

$$G_p(s) = \frac{b_{cp}s^c + b_{(c-1)p}s^{c-1} + \ldots + b_{1p}s + b_{0p}}{s^n + a_{(n-1)p}s^{n-1} + a_{(n-2)p}s^{n-2} + \ldots + a_{1p}s + a_{0p}} \qquad \text{(IV-1)}$$

where $c \leq n - 1$. The coefficients $a_{0p}, \ldots, a_{n-1p}$ and $b_{0p}, \ldots, b_{cp}$ are assumed to be constant, unknown parameters and the polynomial $\sum_{i=0}^{c} b_{ip} s^i$ is assumed to have no zeros in the right-half plane. The model transfer function is assumed to be represented by:

$$G_m(s) = \frac{b_{(n-1)m}s^{n-1} + b_{(n-2)m}s^{n-2} + \ldots + b_{1m}s + b_{0m}}{s^n + a_{(n-1)m}s^{n-1} + a_{(n-2)m}s^{n-2} + \ldots + a_{1m}s + a_{0m}} \qquad \text{(IV-2)}$$

The coefficients $a_{0m}, \ldots, a_{(n-1)m}$ and $b_{0m}, \ldots, b_{(n-1)m}$ are constants and the model transfer function is assumed to be a minimum-phase network. The response error in the frequency-domain is defined by the equation:

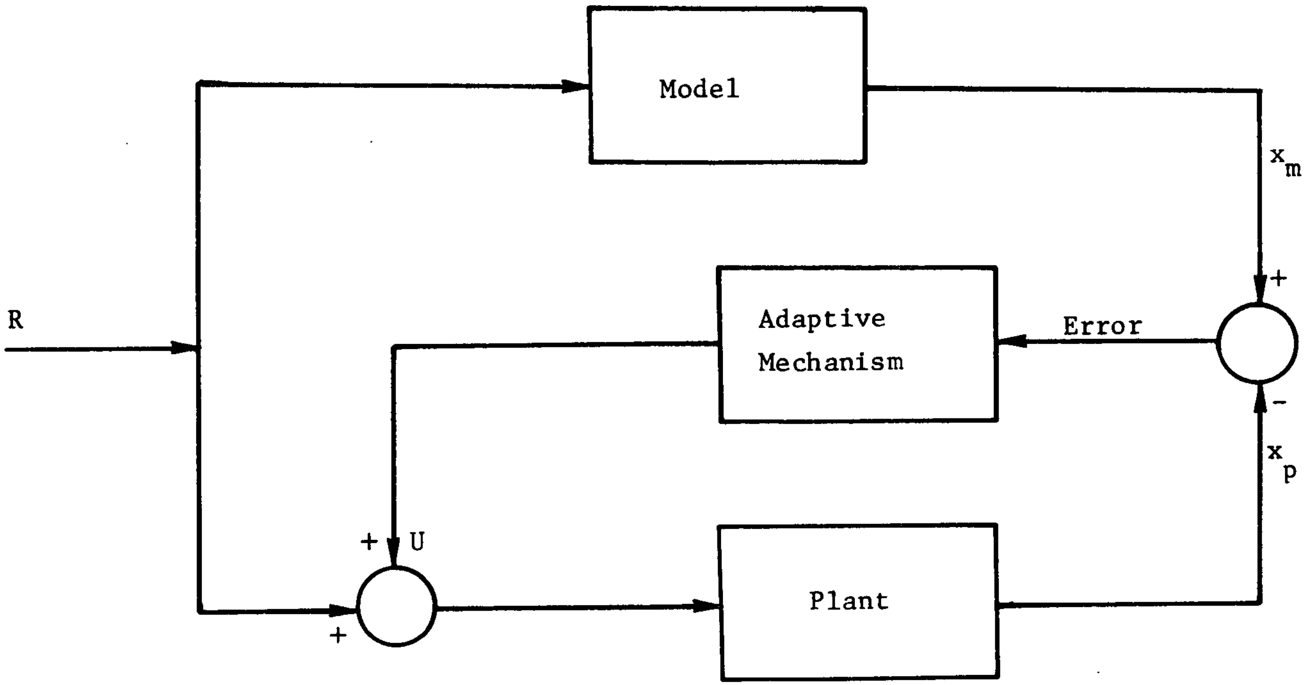
$$E(s) \triangleq X_m(s) - X_p(s) \tag{IV-3}$$

Thus, the object of the frequency-domain adaptive control research was to develope a design procedure for determining pre-filter and feedback filters with characteristics directly related to the desired model in such a manner that the response error would be asymptotically stable.

B. Development of Pre-Filters and Feedback Filters for General Fourth Order System

In order to fully illustrate the design procedure the technique will be developed in detail for a general fourth order system and then extended to a general n^{th} order system. The basic system block diagram is shown in Figure IV-1. The additional input, U, which is the sum of the outputs of the pre-filters and feedback filters, is to be designed so that the plant will track the model. The general fourth order plant is assumed to be represented in the frequency-domain by:

$$G_p(s) = \frac{b_{3p}s^3 + b_{2p}s^2 + b_{1p}s + b_{0p}}{s^4 + a_{3p}s^3 + a_{2p}s^2 + a_{1p}s + a_{0p}} \tag{IV-4}$$



IV-1. Basic system block diagram for frequency-domain technique.

The model is also assumed to be represented in the frequency domain by the relationship:

$$G_m(s) = \frac{b_{3m}s^3 + b_{2m}s^2 + b_{1m}s + b_{0m}}{s^4 + a_{3m}s^3 + a_{2m}s^2 + a_{1m}s + a_{0m}} \qquad \text{(IV-5)}$$

Using Equations (IV-3), (IV-4), and (IV-5), the following expression for the response error may be obtained.

$$\begin{aligned} &E(s)(s^4 + a_{3m}s^3 + a_{2m}s^2 + a_{1m}s + a_{0m}) = \\ &(a_{3p} - a_{3m})s^3X_p(s) + (a_{2p} - a_{2m})s^2X_p(s) + (a_{1p} - a_{1m})sX_p(s) \\ &+ (a_{0p} - a_{0m})X_p(s) + (b_{3m} - b_{3p})s^3R(s) + (b_{2m} - b_{2p})s^2R(s) \\ &+ (b_{1m} - b_{1p})sR(s) + (b_{0m} - b_{0p})R(s) - b_{3p}s^3U(s) \\ &- b_{2p}s^2U(s) - b_{1p}sU(s) - b_{0p}U(s) \end{aligned} \qquad \text{(IV-6)}$$

This step is more easily illustrated in the time-domain. Equation (IV-4) may be represented in differential equation form as:

$$\frac{d^4 x_p}{dt^4} + a_{3p}\frac{d^3 x_p}{dt^3} + a_{2p}\frac{d^2 x_p}{dt^2} + a_{1p}\frac{dx_p}{dt} + a_{0p}x_p =$$

$$b_{3p}\left[\frac{d^3R}{dt^3} + \frac{d^3U}{dt^3}\right] + b_{2p}\left[\frac{d^2R}{dt^2} + \frac{d^2U}{dt^2}\right] + b_{1p}\left[\frac{dR}{dt} + \frac{dU}{dt}\right]$$

$$+ b_{0p}[R + U] \qquad \text{(IV-7)}$$

Likewise, in the time-domain, Equation (IV-5) may be represented as:

$$\frac{d^4x_m}{dt^4} + a_{3m}\frac{d^3x_m}{dt^3} + a_{2m}\frac{d^2x_m}{dt^2} + a_{1m}\frac{dx_m}{dt} + a_{0m}x_m =$$

$$b_{3m}\frac{d^3R}{dt^3} + b_{2m}\frac{d^2R}{dt^2} + b_{1m}\frac{dR}{dt} + b_{0m}R \qquad \text{(IV-8)}$$

By subtracting Equation (IV-7) from (IV-8), adding and subtracting identical terms, the following expression may be obtained:

$$\frac{d^4x_m}{dt^4} - \frac{d^4x_p}{dt^4} + a_{3m}\left[\frac{d^3x_m}{dt^3} - \frac{d^3x_p}{dt^3}\right]$$

$$+ a_{2m}\left[\frac{d^2x_m}{dt^2} - \frac{d^2x_p}{dt^2}\right] + a_{1m}\left[\frac{dx_m}{dt} - \frac{dx_p}{dt}\right]$$

$$+ a_{0m}(x_m - x_p) = (a_{3p} - a_{3m})\frac{d^3x_p}{dt^3} + (a_{2p} - a_{2m})\frac{d^2x_p}{dt^2}$$

$$+ (a_{1p} - a_{1m})\frac{dx_p}{dt} + (a_{0p} - a_{0m})x_p + (b_{3m} - b_{3p})\frac{d^3R}{dt^3}$$

$$+ (b_{2m} - b_{2p})\frac{d^2R}{dt^2} + (b_{1m} - b_{1p})\frac{dR}{dt} + (b_{0m} - b_{0p})R$$

$$- b_{3p}\frac{d^3U}{dt^3} - b_{2p}\frac{d^2U}{dt^2} - b_{1p}\frac{dU}{dt} - b_{0p}U \qquad \text{(IV-9)}$$

Thus, taking the Laplace transform of the preceding expression gives Equation (IV-6).

Let us now make the following definitions:

$$\alpha_1 = (a_{3p} - a_{3m}),\ \alpha_2 = (a_{2p} - a_{2m}),\ \alpha_3 = (a_{1p} - a_{1m})$$

$$\alpha_4 = (a_{0p} - a_{0m}),\ \alpha_5 = (b_{3m} - b_{3p}),\ \alpha_6 = (b_{2m} - b_{2p})$$

$$\alpha_7 = (b_{1m} - b_{1p}),\ \alpha_8 = (b_{0m} - b_{0p}),\ \alpha_9 = b_{3p}$$

$\alpha_{10} = -b_{2p}$, $\alpha_{11} = -b_{1p}$, $\alpha_{12} = -b_{0p}$, and

$$\Delta = s^4 + a_{3m}s^3 + a_{2m}s^2 + a_{1m}s + a_{0m} \qquad \text{(IV-10)}$$

Multiplying Equation (IV-6) by s, using the definitions of Equation (IV-10) and rearranging terms, the following expresssion may be obtained.

$$sE(s) = \frac{X_p(s)}{\Delta}(\alpha_1 s^4 + \alpha_2 s^3 + \alpha_3 s^2 + \alpha_4 s)$$

$$+ \frac{R(s)}{\Delta}(\alpha_5 s^4 + \alpha_6 s^3 + \alpha_7 s^2 + \alpha_8 s)$$

$$- \alpha_9 U(s) + \frac{U(s)}{\Delta}[(a_{3m}\alpha_9 + \alpha_{10})s^3 + (a_{2m}\alpha_9 + \alpha_{11})s^2$$

$$+ (a_{1m}\alpha_9 + \alpha_{12})s + a_{0m}] \qquad \text{(IV-11)}$$

The terms involving a_{0m}, a_{1m}, a_{2m}, a_{3m}, and U(s) result from dividing the expression s^4 by Δ. The time-domain expression for the time derivative of the error may be found by taking the inverse Laplace Transform of Equation (IV-11). Doing so gives the expression:

$$\dot{e}(t) = \alpha_1 \mathcal{L}^{-1}\{\frac{s^4 X_p(s)}{\Delta}\} + \alpha_2 \mathcal{L}^{-1}\{\frac{s^3 X_p(s)}{\Delta}\} + \alpha_3 \mathcal{L}^{-1}\{\frac{s^2 X_p(s)}{\Delta}\}$$

$$+ \alpha_4 \mathcal{L}^{-1}\{\frac{sX_p(s)}{\Delta}\} + \alpha_5 \mathcal{L}^{-1}\{\frac{s^4 R(s)}{\Delta}\} + \alpha_6 \mathcal{L}^{-1}\{\frac{s^3 R(s)}{\Delta}\} + \alpha_7 \mathcal{L}^{-1}\{\frac{s^2 R(s)}{\Delta}\}$$

$$+ \alpha_8 \mathcal{L}^{-1}\{\frac{sR(s)}{\Delta}\}$$

$$- \alpha_9 u(t) + \alpha_9 \mathcal{L}^{-1}\{\frac{(a_{3m}s^3 + a_{2m}s^2 + a_{1m}s + a_{0m})}{\Delta} U(s)\}$$

$$+ \alpha_{10} \mathcal{L}^{-1}\{\frac{s^3U(s)}{\Delta}\} + \alpha_{11} \mathcal{L}^{-1}\{\frac{s^2U(s)}{\Delta}\} + \alpha_{12} \mathcal{L}^{-1}\{\frac{sU(s)}{\Delta}\} \qquad \text{(IV-12)}$$

Let us now select u(t) as:

$$u(t) = \sum_{i=1}^{12} K_i Z_i \qquad \text{(IV-13)}$$

where K_i, i = 1,..., 12 are filter gains to be determined via Liapunov's direct method and Z_i, i = 1,..., 12 are given by the expression:

$$Z_1 = \mathcal{L}^{-1}\{\frac{s^4X_p(s)}{\Delta}\} \quad , \quad Z_2 = \mathcal{L}^{-1}\{\frac{s^3X_p(s)}{\Delta}\}$$

$$Z_3 = \mathcal{L}^{-1}\{\frac{s^2X_p(s)}{\Delta}\} \quad , \quad Z_4 = \mathcal{L}^{-1}\{\frac{sX_p(s)}{\Delta}\}$$

$$Z_5 = \mathcal{L}^{-1}\{\frac{s^4R(s)}{\Delta}\} \quad , \quad Z_6 = \mathcal{L}^{-1}\{\frac{s^3R(s)}{\Delta}\}$$

$$Z_7 = \mathcal{L}^{-1}\{\frac{s^2R(s)}{\Delta}\} \quad , \quad Z_8 = \mathcal{L}^{-1}\{\frac{sR(s)}{\Delta}\}$$

$$Z_9 = \mathcal{L}^{-1}\{\frac{(a_{3m}s^3 + a_{2m}s^2 + a_{1m}s + a_{0m})\ U(s)}{\Delta}\}$$

$$Z_{10} = \mathcal{L}^{-1}\{\frac{s^3U(s)}{\Delta}\} \quad , \quad Z_{11} = \mathcal{L}^{-1}\{\frac{s^2U(s)}{\Delta}\}$$

$$Z_{12} = \mathcal{L}^{-1}\{\frac{sU(s)}{\Delta}\} \tag{IV-14}$$

Thus, Equation (IV-12) may be expressed as

$$\dot{e}(t) - (\alpha_1 - K_1\alpha_9)Z_1 + (\alpha_2 - K_2\alpha_9)Z_2 + (\alpha_3 - K_3\alpha_9)Z_3$$
$$+ (\alpha_4 - K_4\alpha_9)Z_4 + (\alpha_5 - K_5\alpha_9)Z_5 + (\alpha_6 - K_6\alpha_9)Z_6$$
$$+ (\alpha_7 - K_7\alpha_9)Z_7 + (\alpha_8 - K_8\alpha_9)Z_8 + (\alpha_9 - K_9\alpha_9)Z_9$$
$$+ (\alpha_{10} - K_{10}\alpha_9)Z_{10} + (\alpha_{11} - K_{11}\alpha_9)Z_{11} + (\alpha_{12} - K_{12}\alpha_9)Z_{12} \tag{IV-15}$$

Equation (IV-15) may also be expressed by the summation:

$$\dot{e}(t) = \sum_{i=1}^{12}(\alpha_i - K_i\alpha_9)Z_i \tag{IV-16}$$

Thus, Equation (IV-13) gives the expression for the auxiliary input to the system which is a function of the pre-filter and feedback filter networks defined in Equation (IV-14). Equation (IV-16) gives the expression for the time derivative of the error as a function of the pre-filters and feedback filters and their gains. It is necessary, then, using Liapunov's direct method to obtain an expression for the filter gains K_i, $i = 1,\ldots, 12$ in such a manner that the response error is asymptotically stable.

Let us select a Liapunov function of the following form:

$$V = e^2 + \sum_{i=1}^{12} \frac{1}{\beta_i}[(\alpha_i - K_i\alpha_9) + \delta_i(eZ_i)]^2 \tag{IV-17}$$

where β_i and δ_i are constants greater than zero. Thus, the V function of Equation (IV-17) is positive definite. The time derivative of Equation (IV-17) is given by the relationship:

$$\dot{V} = 2e\dot{e} + 2\sum_{i=1}^{12}\frac{1}{\beta_i}[(\alpha_i - K_i\alpha_9) + \delta_i(eZ_i)] \cdot$$

$$[-\dot{K}_i\alpha_9 + \delta_i\frac{d}{dt}(eZ_i)] \tag{IV-18}$$

Let us now assign $\dot{K}_i$ the value

$$\dot{K}_i = \frac{\beta_i}{\alpha_9}(eZ_i) + \frac{\delta_i}{\alpha_9}\frac{d}{dt}(eZ_i) \tag{IV-19}$$

Substitution of Equation (IV-17) and (IV-19) into Equation (IV-18) gives:

$$\dot{V} = 2e\sum_{i=1}^{12} (\alpha_i - K_i\alpha_9)Z_i$$

$$+ 2\sum_{i=1}^{12} \frac{1}{\beta_i}[(\alpha_i - K_i\alpha_9) + \delta_i(eZ_i)][-\beta_i(eZ_i)] \tag{IV-20}$$

which may be written as:

$$\dot{V} = -e^2 \sum_{i=1}^{12} Z_i \delta_i^2 \quad . \tag{IV-21}$$

Application of Theorem II-4 thus shows that with the selection of K_i, $i = 1, \ldots, 12$ as given by Equation (IV-19), that the response error is asymptotically stable.

Integrating Equation (IV-19) gives the expression for the filter gains which are:

$$K_i = \sigma_i \int_{t_0}^{t} (eZ_i) d\pi + \rho_i (eZ_i) + K_i(t_o) \qquad i = 1, \ldots, 12 \tag{IV-22}$$

where $\sigma_i = \dfrac{\beta_i}{\alpha_9}$ and $\rho_i = \dfrac{\delta_i}{\alpha_9}$

Equations (IV-14) and (IV-19) then give the necessary relationships for the pre-filters, feedback filters and their gains so that the response error is asymptotically stable.

As may be seen from inspection of Equation (IV-15), the filters may be grouped into 3 filter networks as given by Equation (IV-23). For mathematical convenience, frequency-domain notation will be used although the gains are time-varying quantities.

Filter for $X_p(s)$

$$\frac{K_1 s^4 + K_2 s^3 + K_3 s^2 + K_4 s}{\Delta}$$

Filter for R(s)

$$\frac{K_5 s^4 + K_6 s^3 + K_7 s^2 + K_8 s}{\Delta}$$

Filter for U(s)

$$\frac{(K_9 a_{3m} + K_{10}) s^3 + (K_9 a_{2m} + K_{11}) s^2 + (K_9 a_{1m} + K_{12}) s + K_9 a_{0m}}{\Delta} \tag{IV-23}$$

C. Extension to General n^{th} Order System

The basic system block diagram is as discussed in Section B and is shown in Figure IV-1. The plant and the model are given by Equations (IV-1) and (IV-2), respectively.

Applying the methods of Section B, the expressions for Z_i, i 1,..., 3n are given by:

$$Z_1 = \mathcal{L}^{-1}\{\frac{s^n X_p(s)}{\Delta}\} \quad , \quad Z_2 = \mathcal{L}^{-1}\{\frac{s^{n-1} X_p(s)}{\Delta}\} \quad , \ldots$$

$$Z_n = \mathcal{L}^{-1}\{\frac{s X_p(s)}{\Delta}\} \quad , \quad Z_{n+1} = \mathcal{L}^{-1}\{\frac{s^n R(s)}{\Delta}\} \quad ,$$

$$Z_{n+2} = \mathcal{L}^{-1}\{\frac{s^{n-1} R(s)}{\Delta}\} \quad , \ldots , \quad Z_{2n} = \mathcal{L}^{-1}\{\frac{s R(s)}{\Delta}\} \quad ,$$

$$Z_{2n+1} = \mathcal{L}^{-1}\{\frac{(a_{(n-1)m}s^{n-1} + a_{(n-2)m}s^{n-2} +\ldots+ a_{1m}s + a_{0m})U(s)}{\Delta}\} ,$$

$$Z_{3n} = \mathcal{L}^{-1}\{\frac{sU(s)}{\Delta}\} \qquad \text{(IV-24)}$$

where $\Delta = s^n + a_{(n-1)m}s^{n-1} + a_{(n-2)m}s^{n-2} +\ldots+ a_{1m}s + a_{0m}$.

The filter gains K_i, $i = 1,\ldots, 3n$ are given by the expression:

$$K_i = \sigma_i \int_{t_0}^{t} (eZ_i)d\pi + \rho_i(eZ_i) + K_i(t_o) \qquad \text{(IV-25)}$$

where $\sigma_i = \dfrac{\beta_i}{\alpha_{2n+1}}$ and $\rho_i = \dfrac{\delta_i}{\alpha_{2n+1}}$.

As for the system investigated in Section B, the pre-filters and feedback filters and their gains given in Equations (IV-24) and (IV-25), respectively, may be grouped into 3 filter networks as shown in Equation (IV-26).

Filter for $X_p(s)$

$$\frac{K_1s^n + K_2s^{n-1} +\ldots+ K_{n-1}s^2 + K_ns}{\Delta}$$

Filter for R(s)

$$\frac{K_{n+1}s^n + K_{n+2}s^{n-1} +\ldots+ K_{2n-1}s + K_{2n}s}{\Delta}$$

Filter for U(s)

$$\frac{(K_{2n+1}a_{(n-1)m} + K_{2n+2})s^{n-1} + (K_{2n+1}\, a_{(n-2)m} + K_{2n+3})s^{n-2}}{}$$

$$\frac{+\ldots+ (K_{2n+1}a_{1m} + K_{3n})s + K_{2n+1}a_{0m}}{\Delta} \qquad \text{(IV-26)}$$

The generalized expression for the error equation is given by:

$$\dot{e}(t) = \sum_{i=1}^{3n} (\alpha_i - K_i\alpha_{2n+1})Z_i \qquad \text{(IV-27)}$$

In steady-state, the gains K_i, $i = 1, \ldots, 3n$ become constant and are given by:

$$\lim_{t \to \infty} K_i = \frac{\alpha_i}{\alpha_{2n+1}} \qquad \text{(IV-28)}$$

D. <u>Application of Design Technique</u>

A model of the National Aeronautics and Space Administration's Space Shuttle vehicle is used, as in Chapter III, in order to illustrate the application of the frequency-domain design technique. The design technique is applied to the same basic system configurations as was investigated in Chapter III. The Space Shuttle vehicle is investigated assuming both linear and non-linear characteristics. Simulation studies were made in order to determine the effect of wind gust disturbances on the adaption scheme. In addition, it is shown by simulation results that the adaption scheme may be applied to a time-varying system although the frequency-domain technique itself is not developed for the time-varying case.

The dynamics used for the Space Shuttle vehicle in this study are as given below in the frequency-domain. The bending mode plant and model transfer functions, $G_{Ep}(s)$ and $G_{Em}(s)$, respectively, are given by:

$$G_{Ep}(s) = \frac{X_{Ep}(s)}{R(s) + U_E(s)} = \frac{s}{s + .2s + 4} \tag{IV-29}$$

and

$$G_{Em}(s) = \frac{X_{Em}(s)}{R(s)} = \frac{s}{s^2 + 2s + 4} \tag{IV-30}$$

The transfer functions for the rotational mode plant and model, denoted by $G_{pp}(s)$ and $G_{pm}(s)$, respectively, are given by:

$$G_{pp}(s) = \frac{X_{pp}(s)}{R(s) + U_p(s)} = \frac{s + 1}{s^2} \tag{IV-31}$$

$$G_{pm}(s) = \frac{X_{pm}(s)}{R(s)} = \frac{s + 1}{s^2 + 1.414s + 1} \qquad \text{(IV-32)}$$

For the bending mode, the filters Z_{iE}, i = 1,..., 6, from Equation (IV-24) are:

$$Z_{1E} = \mathcal{L}^{-1}\{\frac{s^2 X_{Ep}(s)}{\Delta_E(s)}\} \quad , \quad Z_{2E} = \mathcal{L}^{-1}\{\frac{sX_{Ep}(s)}{\Delta_E(s)}\}$$

$$Z_{3E} = \mathcal{L}^{-1}\{\frac{s^2 R(s)}{\Delta_E(s)}\} \quad , \quad Z_{4E} = \mathcal{L}^{-1}\{\frac{sR(s)}{\Delta_E(s)}\}$$

$$Z_{5E} = \mathcal{L}^{-1}\{\frac{(2s + 4)U(s)}{\Delta_E(s)}\} \quad , \quad Z_{6E} = \mathcal{L}^{-1}\{\frac{sU(s)}{\Delta_E(s)}\}$$

where $\Delta_E(s) = s^2 + 2s + 4$.

The gains K_{iE}, i = 1,..., 6, assumed to be zero at time zero, are given by the expression:

$$K_{iE} = \sigma_{iE} \int_0^t (e_E Z_{iE})\, d\pi + \rho_{iE}(e_E Z_{iE}) \qquad \text{(IV-33)}$$

where $e_E = x_{Em} - x_{Ep}$.

The filters for $X_{Ep}(s)$, R(s), and $U_E(s)$, from Equation (IV-26) are:

Filters for $X_{Ep}(s)$

3⊕

$$\frac{K_{1E}s^2 + K_{2E}s}{\Delta_E(s)}$$

Filter for R(s)

$$\frac{K_{3E}s^2 + K_{4E}s}{\Delta_E(s)}$$

Filter for $U_E(s)$

$$\frac{(2K_{5E} + K_{6E})s + 4K_{5E}}{\Delta_E(s)} \tag{IV-34}$$

For the rotational mode, the filters Z_{ip}, $i = 1,\ldots, 6$, from Equation (IV-24) are:

$$Z_{1p} = \mathcal{L}^{-1}\{\frac{s^2 Xpp(s)}{\Delta_p(s)}\} \quad , \quad Z_{2p} = \mathcal{L}^{-1}\{\frac{sXpp(s)}{\Delta_p(s)}\} \quad ,$$

$$Z_{3p} = \mathcal{L}^{-1}\{\frac{s^2 R(s)}{\Delta_p(s)}\} \quad , \quad Z_{4p} = \mathcal{L}^{-1}\{\frac{sR(s)}{\Delta_p(s)}\} \quad ,$$

$$Z_{5p} = \mathcal{L}^{-1}\{\frac{(1.414s+1)U(s)}{\Delta_p(s)}\} \quad , \quad Z_{6p} = \mathcal{L}^{-1}\{\frac{sU(s)}{\Delta_p(s)}\} \tag{IV-35}$$

where $\Delta_p(s) = s^2 + 1.414s + 1$

The gains K_{ip}, $i = 1, \ldots, 6$, assumed to be zero at time zero, are given by the expression:

$$K_{ip} = \sigma_{ip} \int_0^t (e_p Z_{ip}) d\pi + \rho_{ip}(e_p Z_{ip}) \qquad \text{(IV-36)}$$

where $e_p = x_{pm} - x_{pp}$

The filters for $X_{pp}(s)$, $R(s)$, and $U_p(s)$, from Equation (IV-26) are:

Filter for $X_{pp}(s)$

$$\frac{K_{1p}s^2 + K_{2p}s}{\Delta_p(s)}$$

Filter for $R(s)$

$$\frac{K_{3p}s^2 + K_{4p}s}{\Delta_p(s)}$$

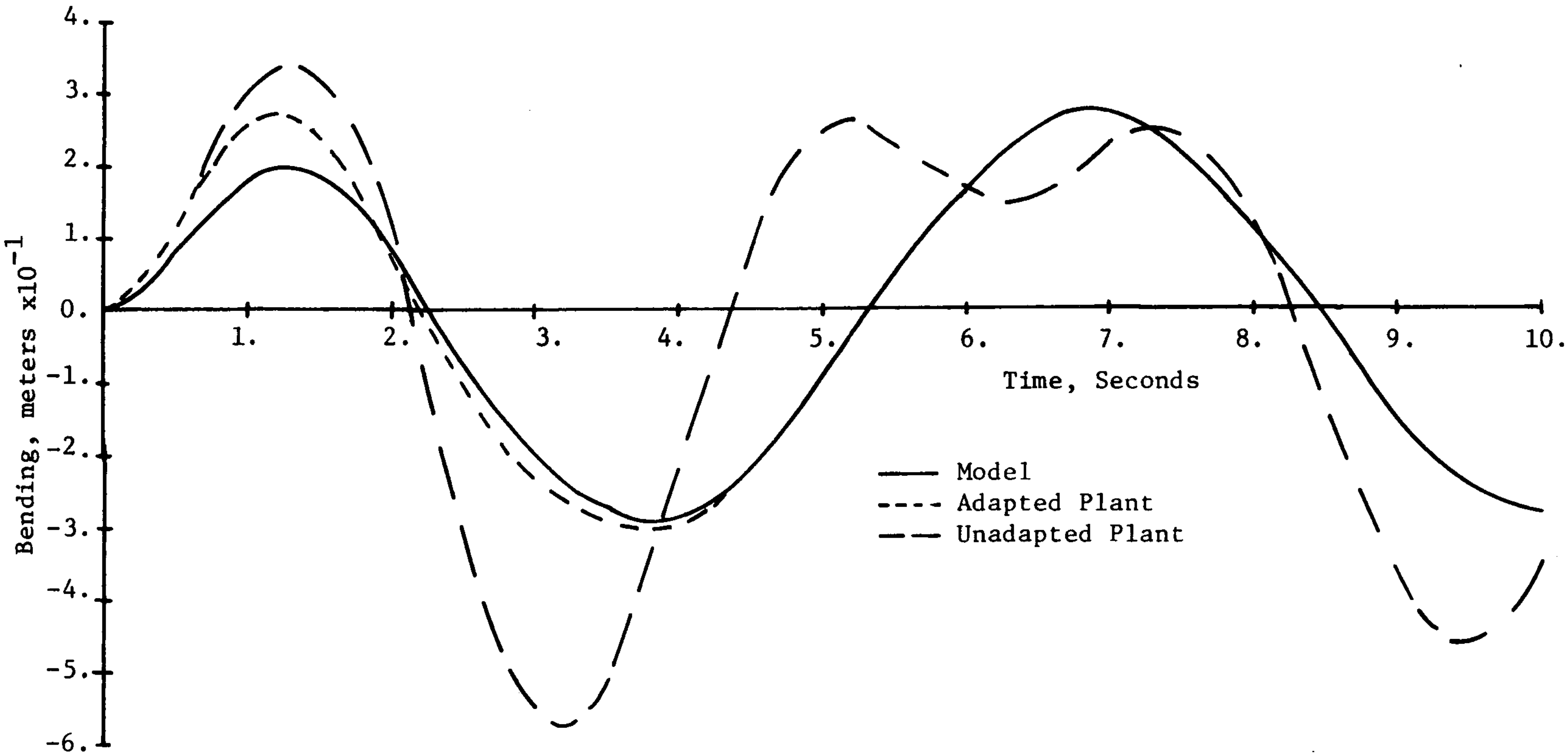
Filter for $U_p(s)$

$$\frac{(1.414K_{5p} + K_{6p})s + K_{5p}}{\Delta_p(s)} \qquad \text{(IV-37)}$$

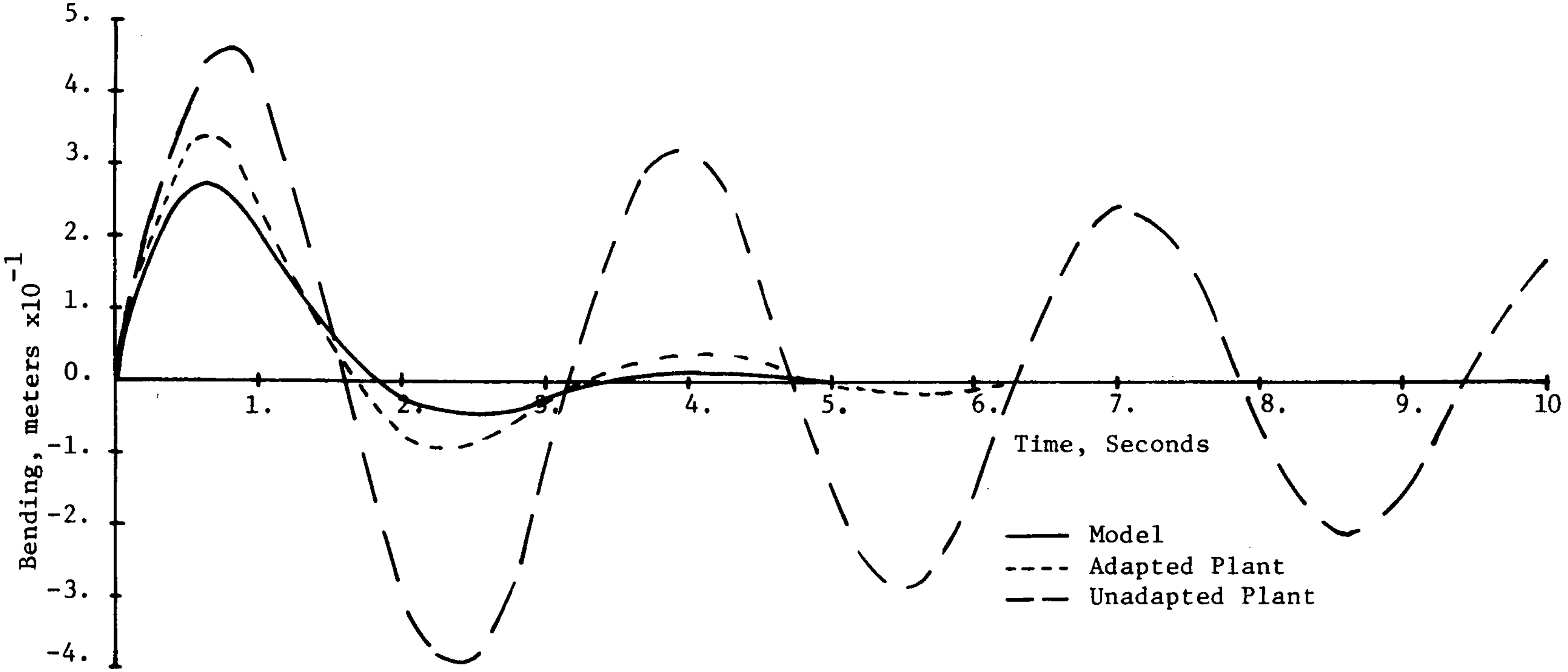
The system with pre-filters and feedback filters was simulated as in Chapter III with CSMP. Two system inputs were investigated for the linear case, the first being a sine wave input and the second a step input. Figure IV-2 shows the bending response of the unadapted plant, the adapted plant, and the model for a sine input to the system. The gains σ_{iE} and ρ_{iE}, i = 1,..., 6 are 20. and 30., respectively. Figure IV-3 shows the bending response of the unadapted plant, the adapted plant, and the model response with a step input to the system. The gains σ_{iE} and ρ_{iE} are as in the previous figure. As can be seen from these figures, the adaptation is rapid and the plant tracks the model very closely.

Figures IV-4 and IV-5 show the rotational position and velocity response of the unadapted plant, the adapted plant, and the model for a sine wave input to the system. For these figures, the gains σ_{ip} and ρ_{ip}, i = 1,..., 6 of Equation (IV-36) are 20. and 30., respectively. Figures IV-6 and IV-7 show the same responses except with the system having a step input. As can be seen from the figures, the response error is very small for both a sine wave input and a step input. The adapted plant and model responses are virtually indistinguishable for the rotational position and velocity for the system with either a sine wave input or a step input.

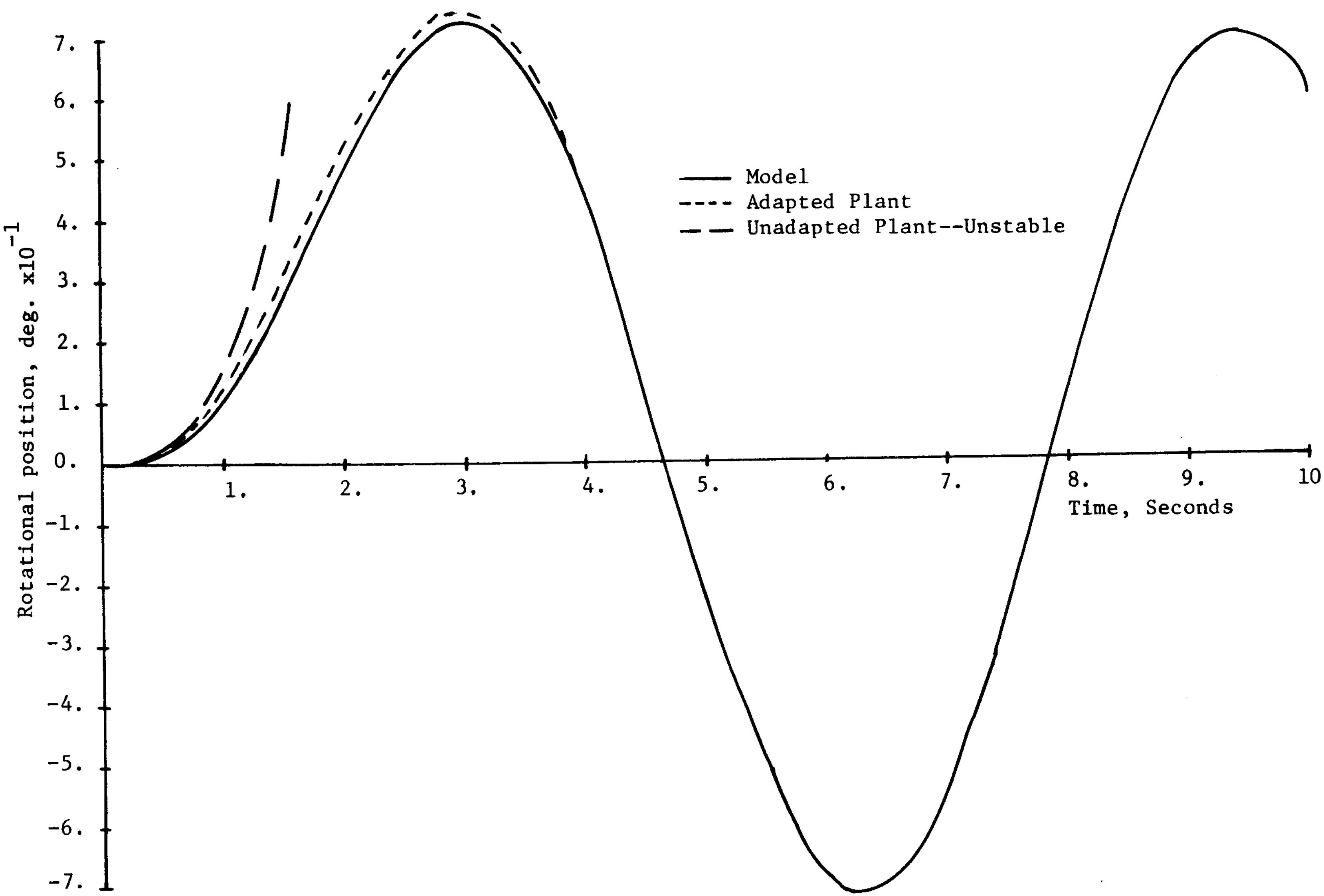
The design technique developed in this chapter was not developed for a system with time-varying parameters. However, a simulation study was made in order to determine if the design technique would provide



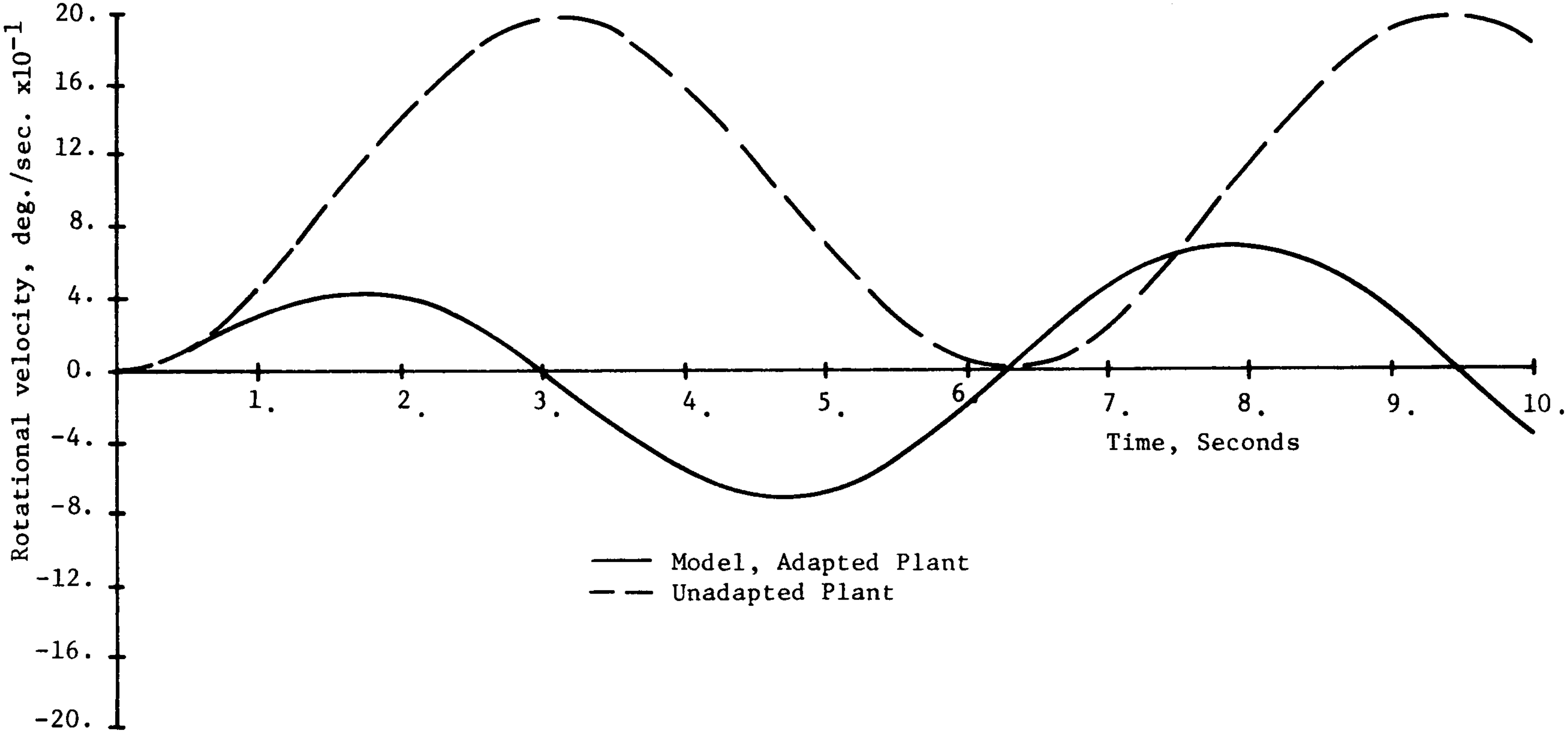
IV-2. Bending mode responses of model, adapted plant, and unadapted plant, sine wave input, frequency-domain technique.



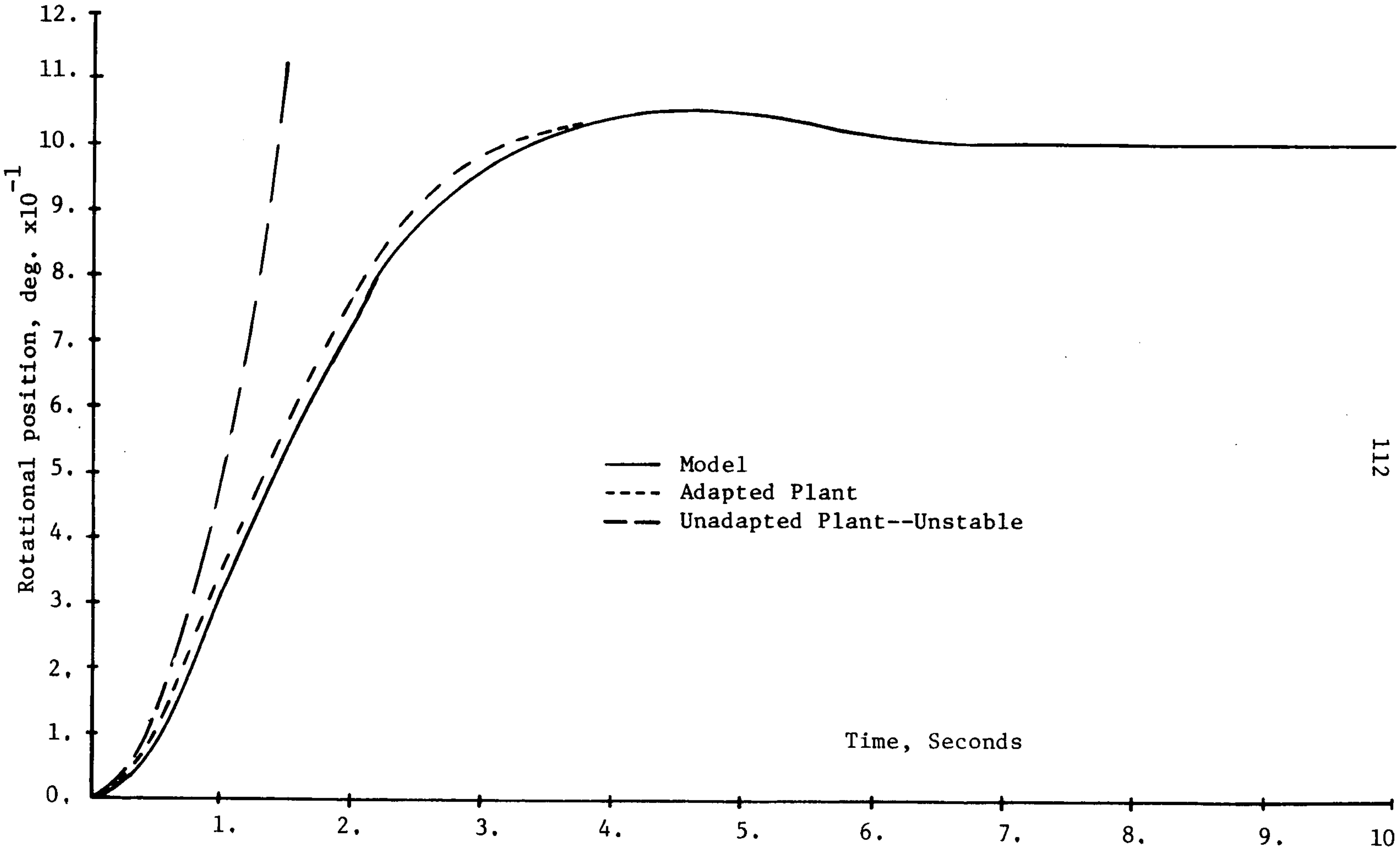
IV-3. Bending mode responses of model, adapted plant, and unadapted plant, step input, frequency-domain technique.



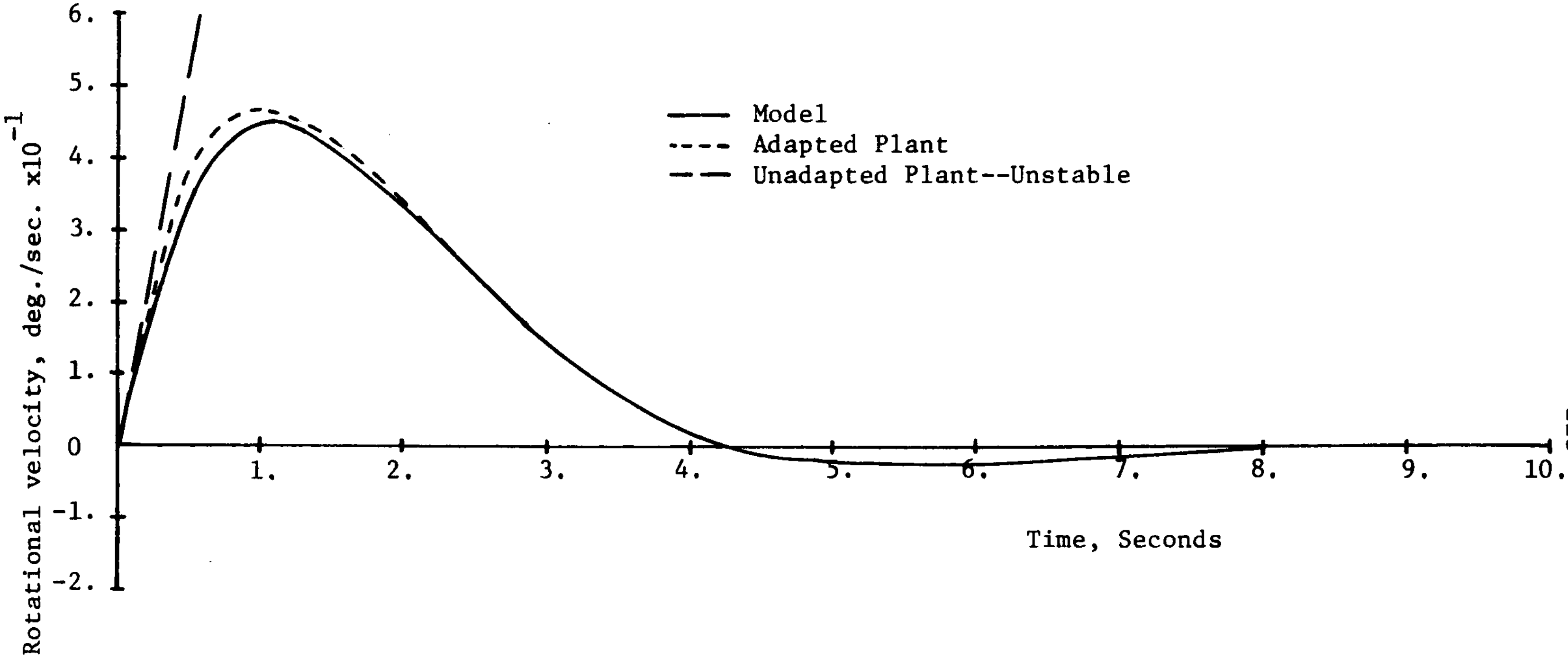
IV-4. Rotational position responses of model, adapted plant, and unadapted plant, sine wave input, frequency-domain technique.



IV-5. Rotational velocity responses of model, adapted plant, and unadapted plant, sine wave input, frequency-domain technique.



IV-6. Rotational position responses of model, adapted plant, and unadapted plant, step input, frequency-domain technique.



IV-7. Rotational velocity responses of model, adapted plant, and unadapted plant, step input, frequency-domain technique.

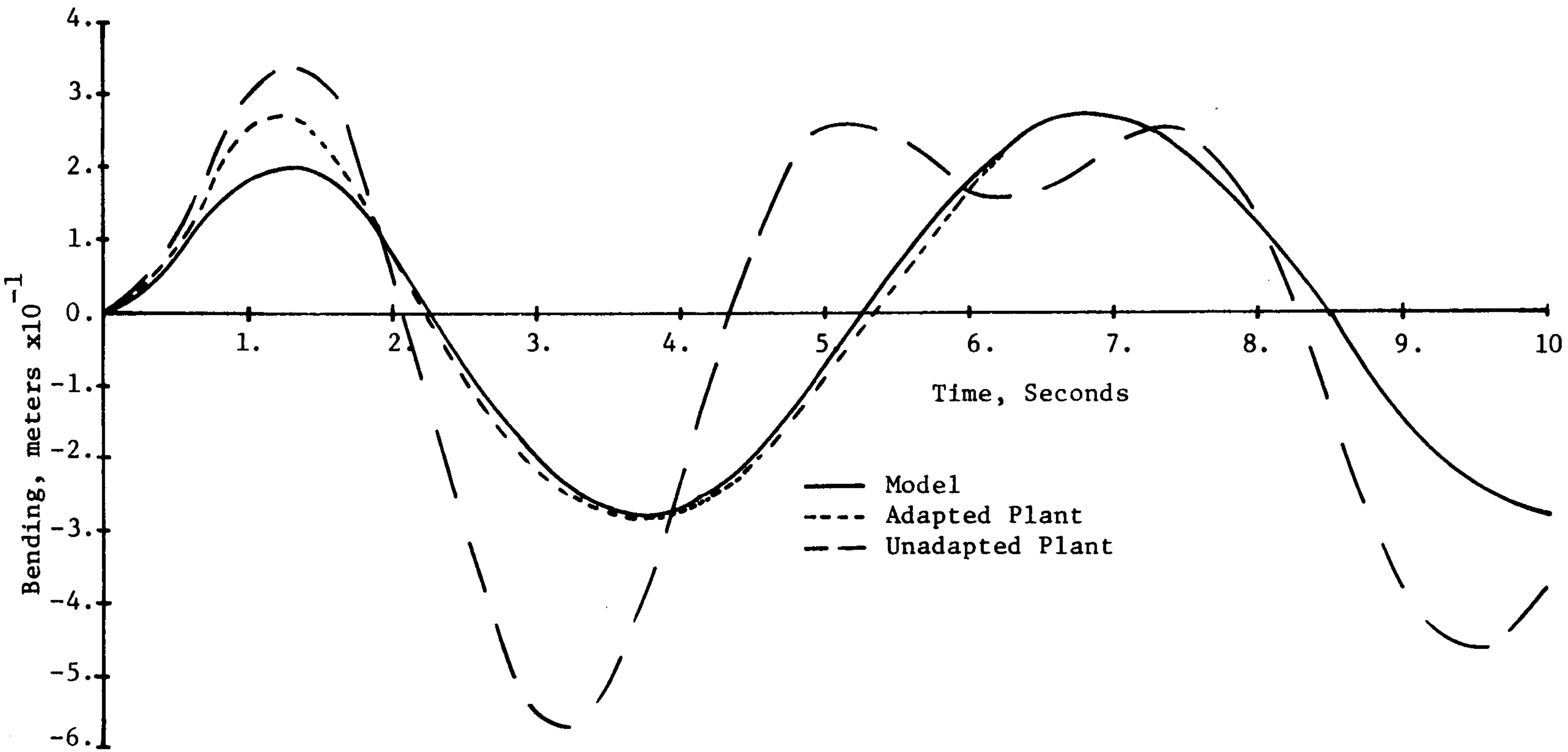
adaptation for a system with some time-varying parameters. For the bending mode, the plant was assumed to be represented by:

$$G_{Ep}(s) = \frac{s}{s^2 + C_E s + 4} \tag{IV-38}$$

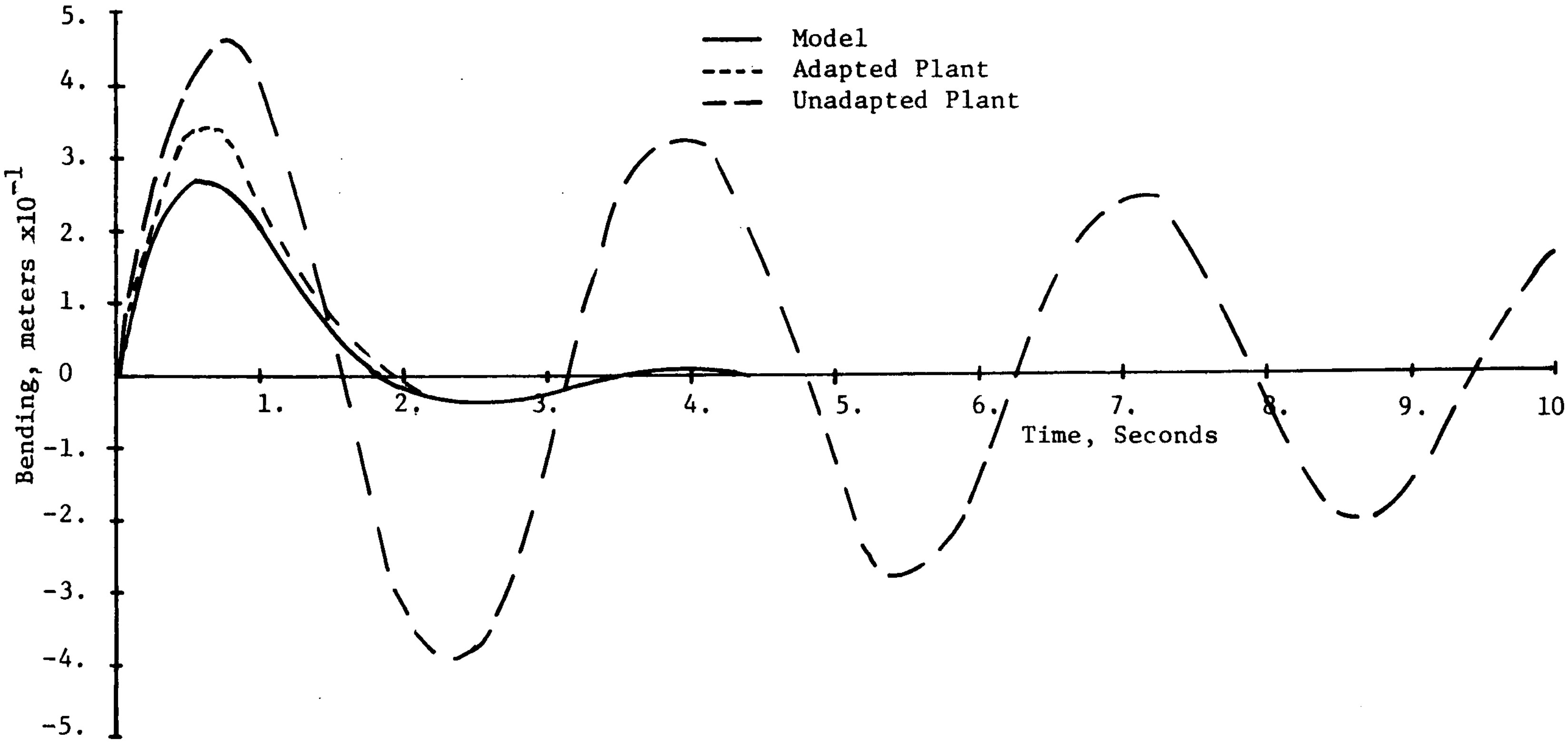
where C_E varies linearly from 0. to .2 in 5. seconds. For the rotational mode, the plant was assumed to be represented by

$$G_{pp}(s) = \frac{s + 1}{s^2 + C_p} \tag{IV-39}$$

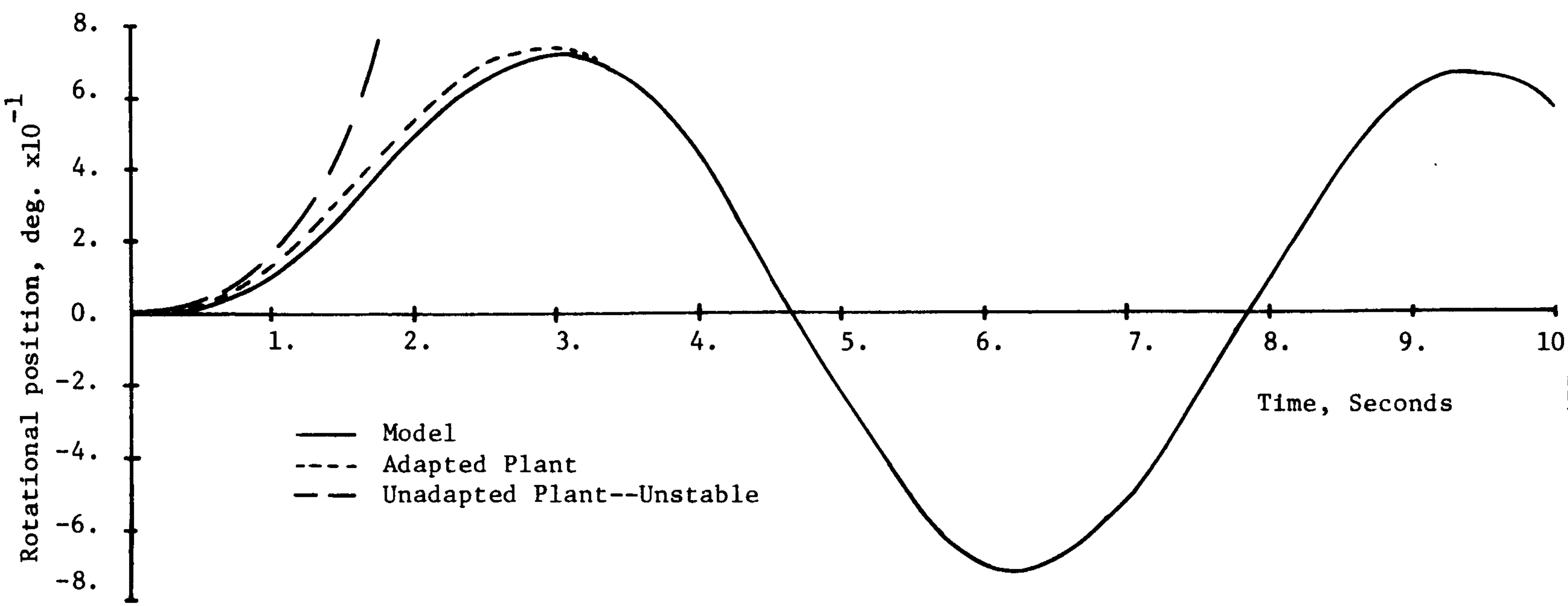
where C_p varies linearly from -.2 to 0. in 5. seconds. For mathematical convenience, frequency-domain notation is used. Figures IV-8 and IV-9 show the bending mode response for the unadapted plant, the adapted plant, and the model for a sine wave input and a step input. As can be seen from the figures, the adaption is not seriously affected and the plant tracks the model very closely. Figures IV-10 and IV-11 show the rotational position and velocity, respectively, of the unadapted plant, the adapted plant, and the model with a sine wave input to the system. Figures IV-12 and IV-13 show the same responses with a step input to the system. The time variations for Figures IV-7 through IV-13 were as noted above. Again, it is seen that the time-varying parameters do not seriously effect the adaptation.



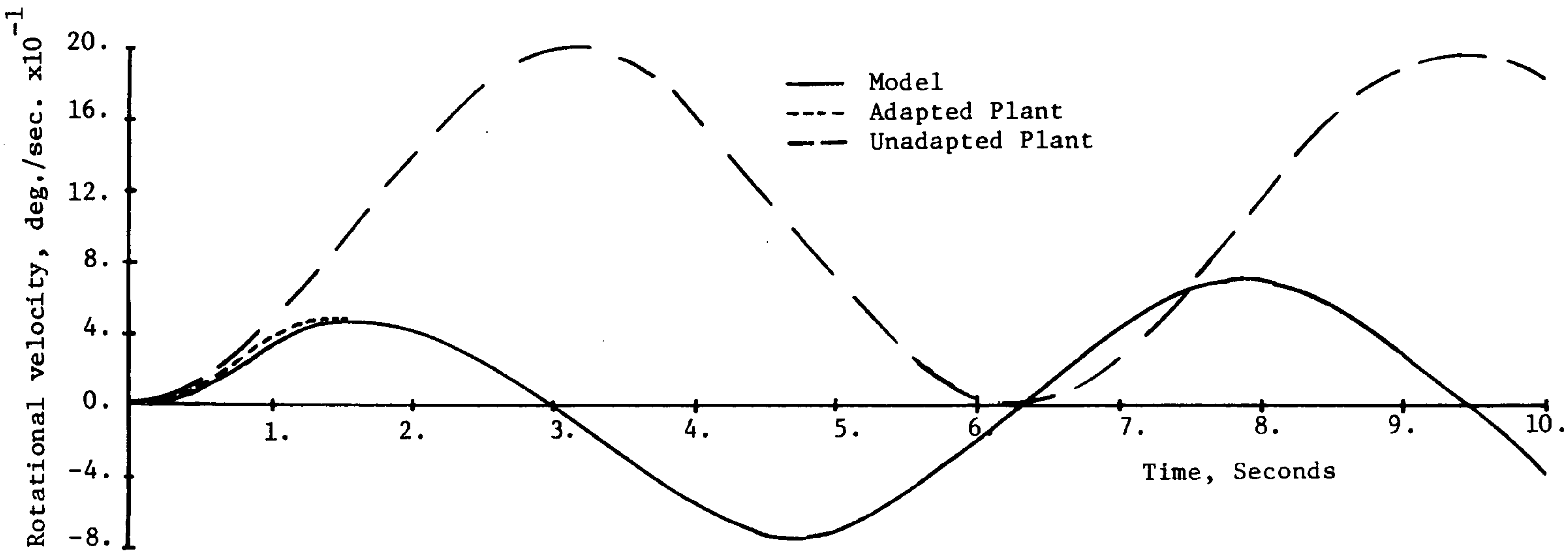
IV-8. Bending mode responses of model, adapted plant, and unadapted plant, sine wave input, time-varying parameters, frequency-domain technique.



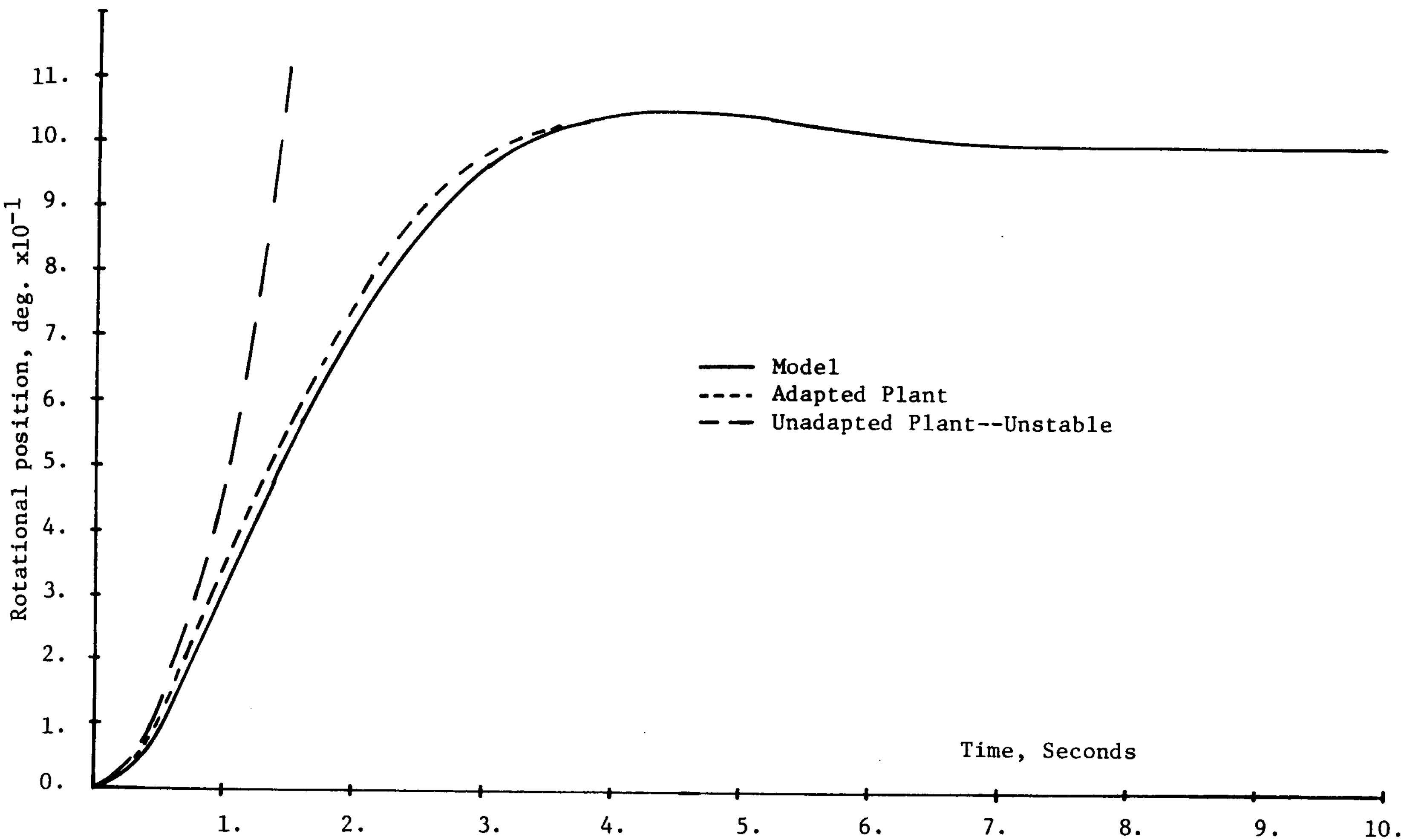
IV-9. Bending mode responses of model, adapted plant, and unadapted plant, step input, time-varying parameters, frequency-domain technique.



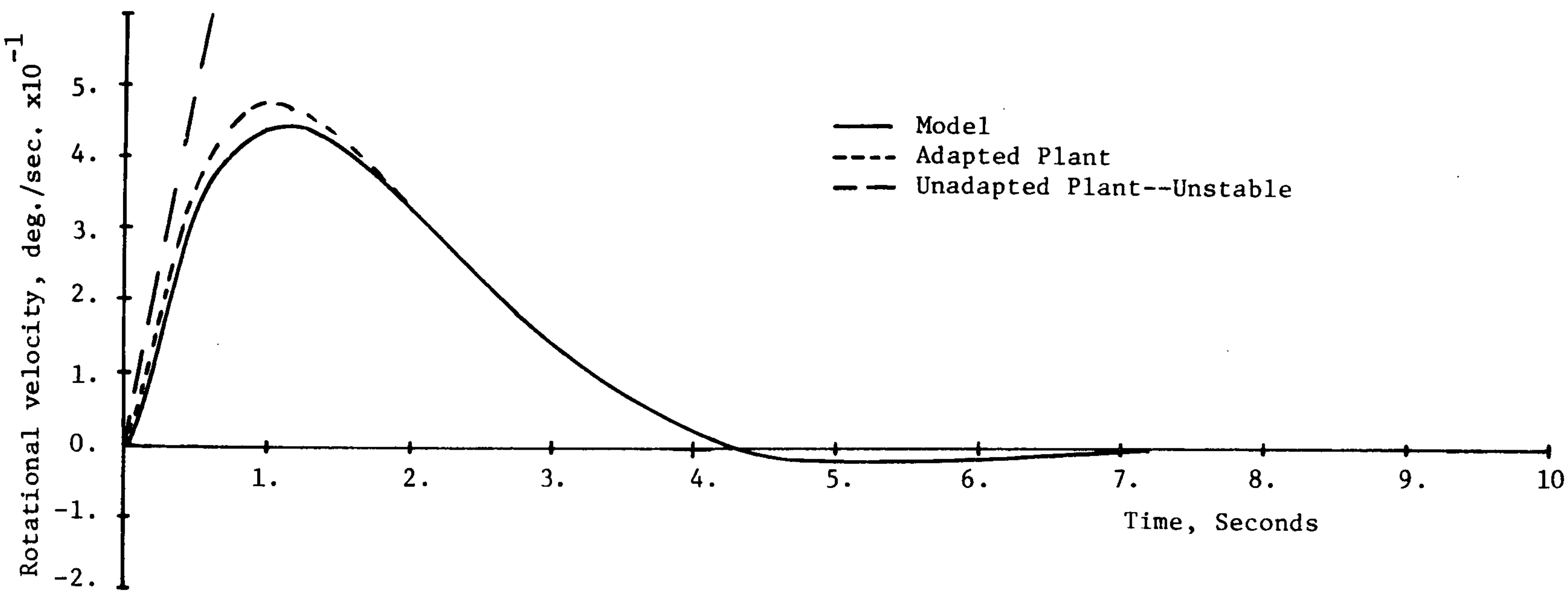
IV-10. Rotational position responses of model, adapted plant, and unadapted plant, sine wave input, time-varying parameters, frequency-domain technique.



IV-11. Rotational velocity responses of model, adapted plant, and unadapted plant, sine wave input, time-varying parameters, frequency-domain technique.



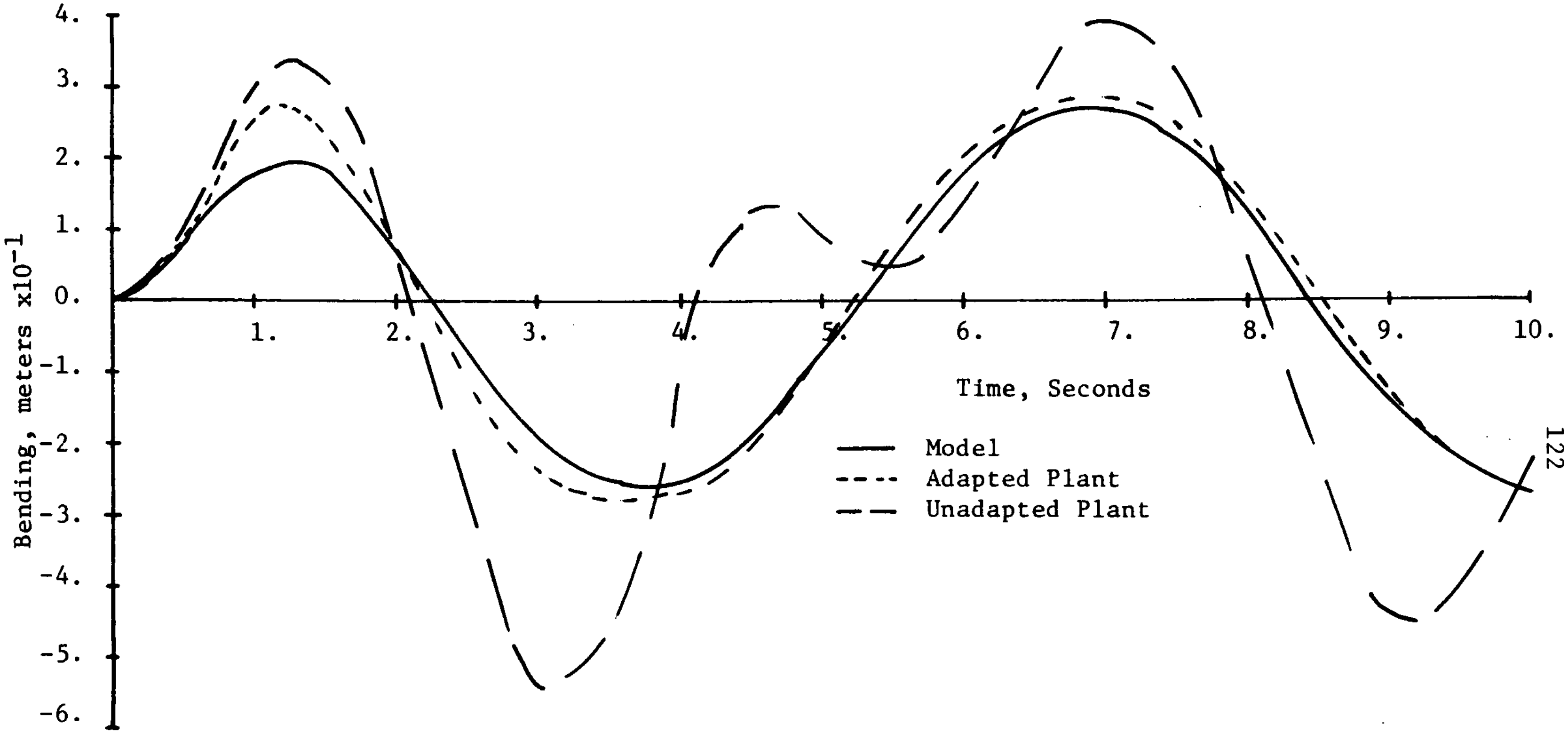
IV-12. Rotational position responses of model, adapted plant, and unadapted plant, step input, time-varying parameters, frequency-domain technique.



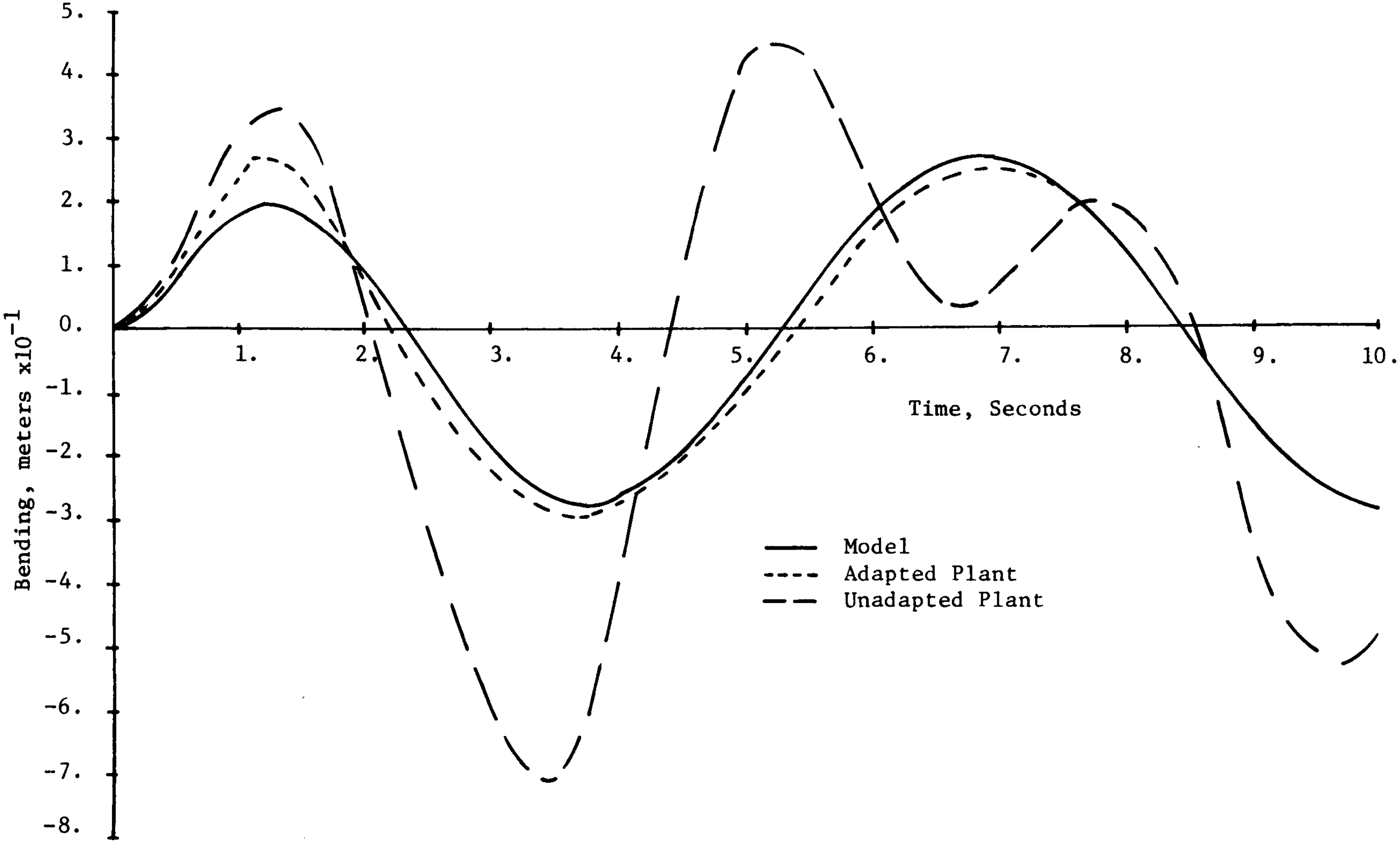
IV-13. Rotational velocity responses of model, adapted plant, and unadapted plant, step input, time-varying parameters, frequency-domain technique.

As was discussed in Section C of Chapter III, one of the inherent capabilities of a "good" adaptive control system is the ability to "adapt" so as to be able to provide the desired response characteristics regardless of disturbances, unknown parameters, etc. With the adaptation of the Space Shuttle being provided via the frequency-domain design technique, a simulation study was performed in order to determine the effects on the vehicle due to wind gusts. As in Chapter III, the wind gusts were simulated by a step input at 3. seconds with a magnitude of 50% of the maximum control force available. Figures IV-14 and IV-15 show the bending mode response of the unadapted plant, the adapted plant, and the model. The responses are shown for a wind gust in the positive direction in Figure IV-14 and in the negative direction in Figure IV-15. The figures show that the adaptation provided by the frequency-domain method compensates successfully for the large disturbances and the plant tracks the model very closely.

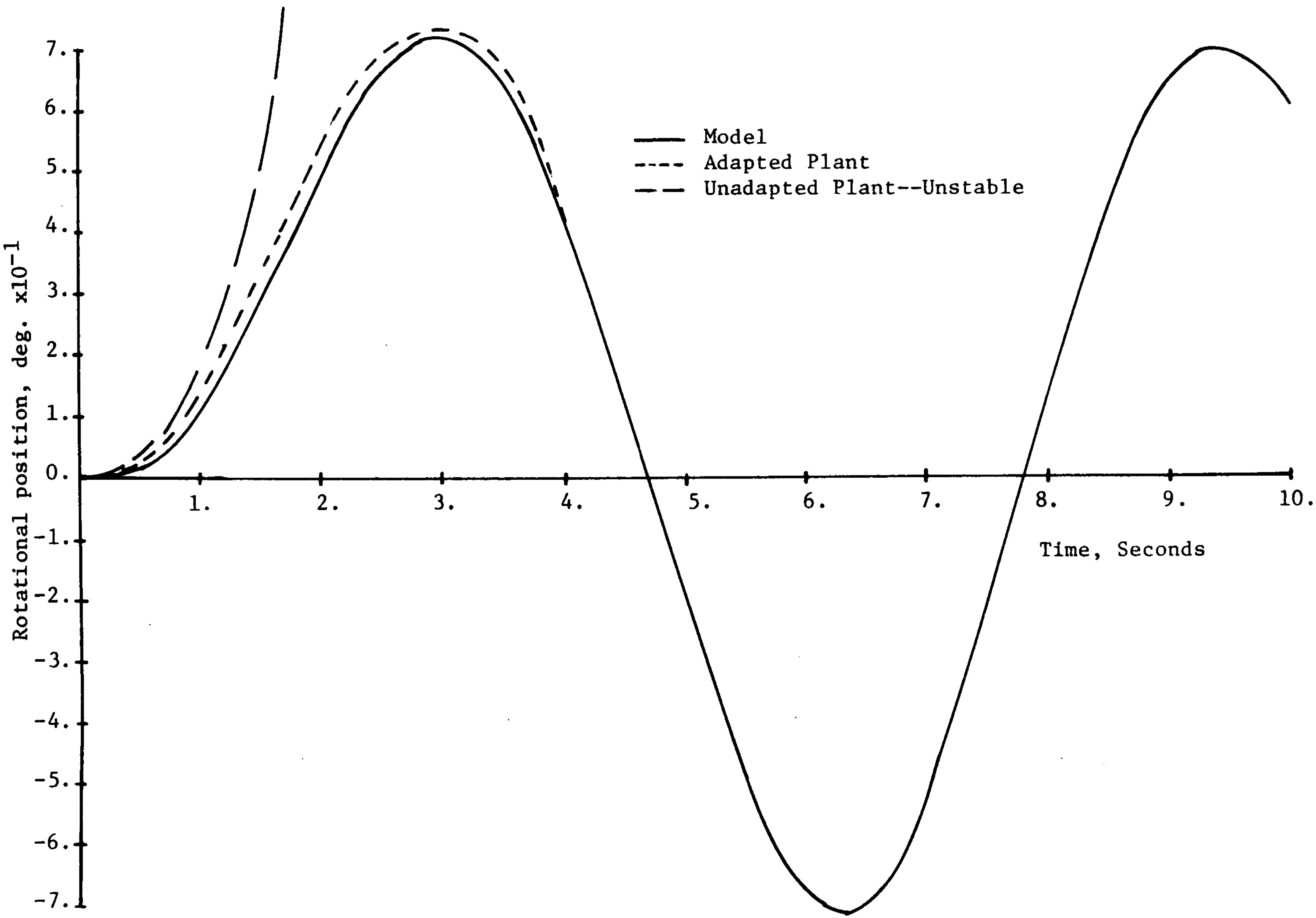
Figures IV-16 and IV-17 show the rotational position responses for the unadapted plant, the adapted plant, and the model for a wind gust in the positive direction and a wind gust in the negative direction, respectively. Figures IV-18 and IV-19 show the rotational velocity responses for the unadapted plant, the adapted plant, and the model for a wind gust in the positive direction and in the negative direction, respectively. As in the case of the bending mode, Figures IV-14 through IV-19 show that the adaptation provided by the frequency-domain design technique compensates very adequately for the wind gust disturbances.



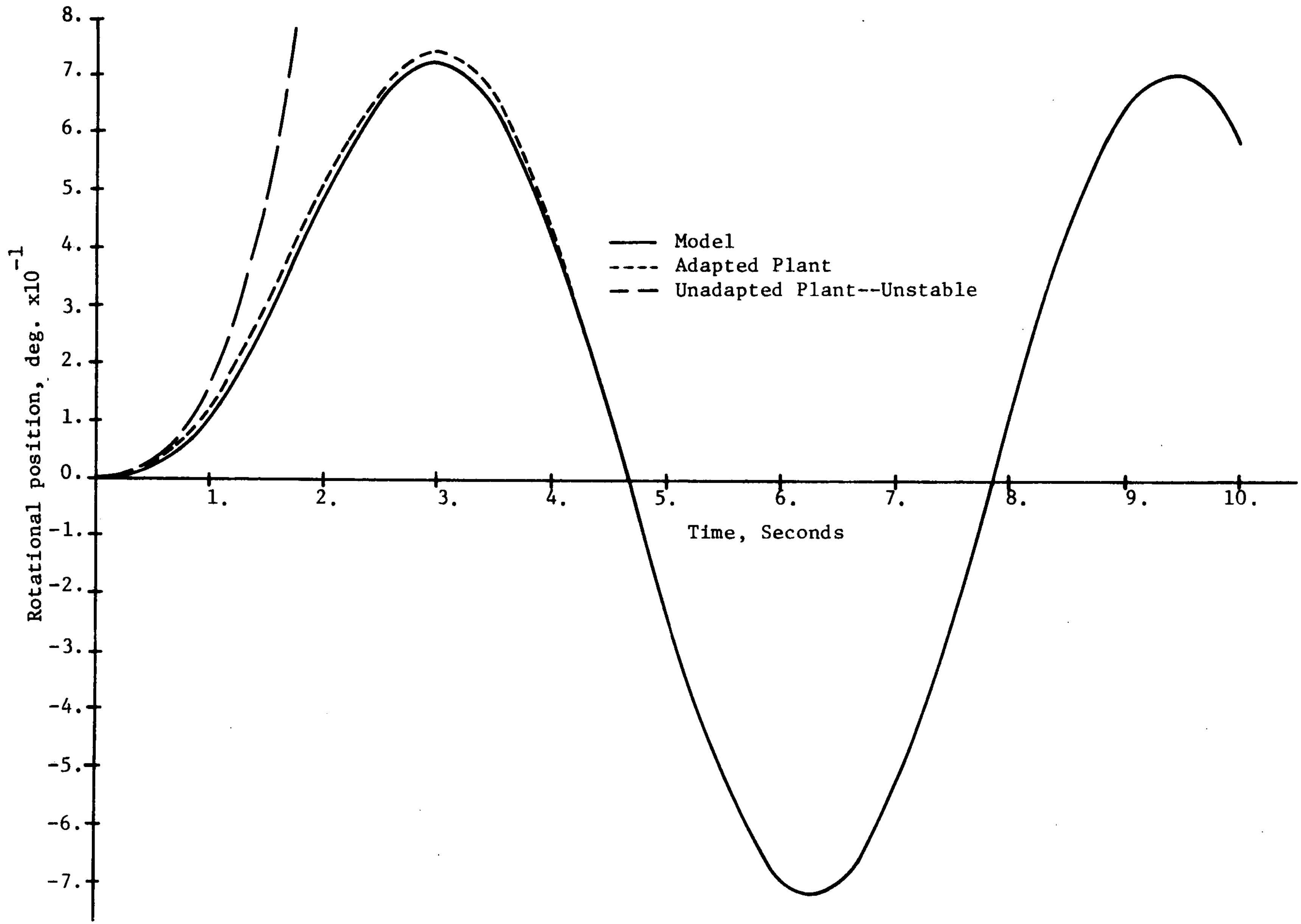
IV-14. Bending mode responses of model, adapted plant, and unadapted plant, negative wind gust, frequency-domain technique.



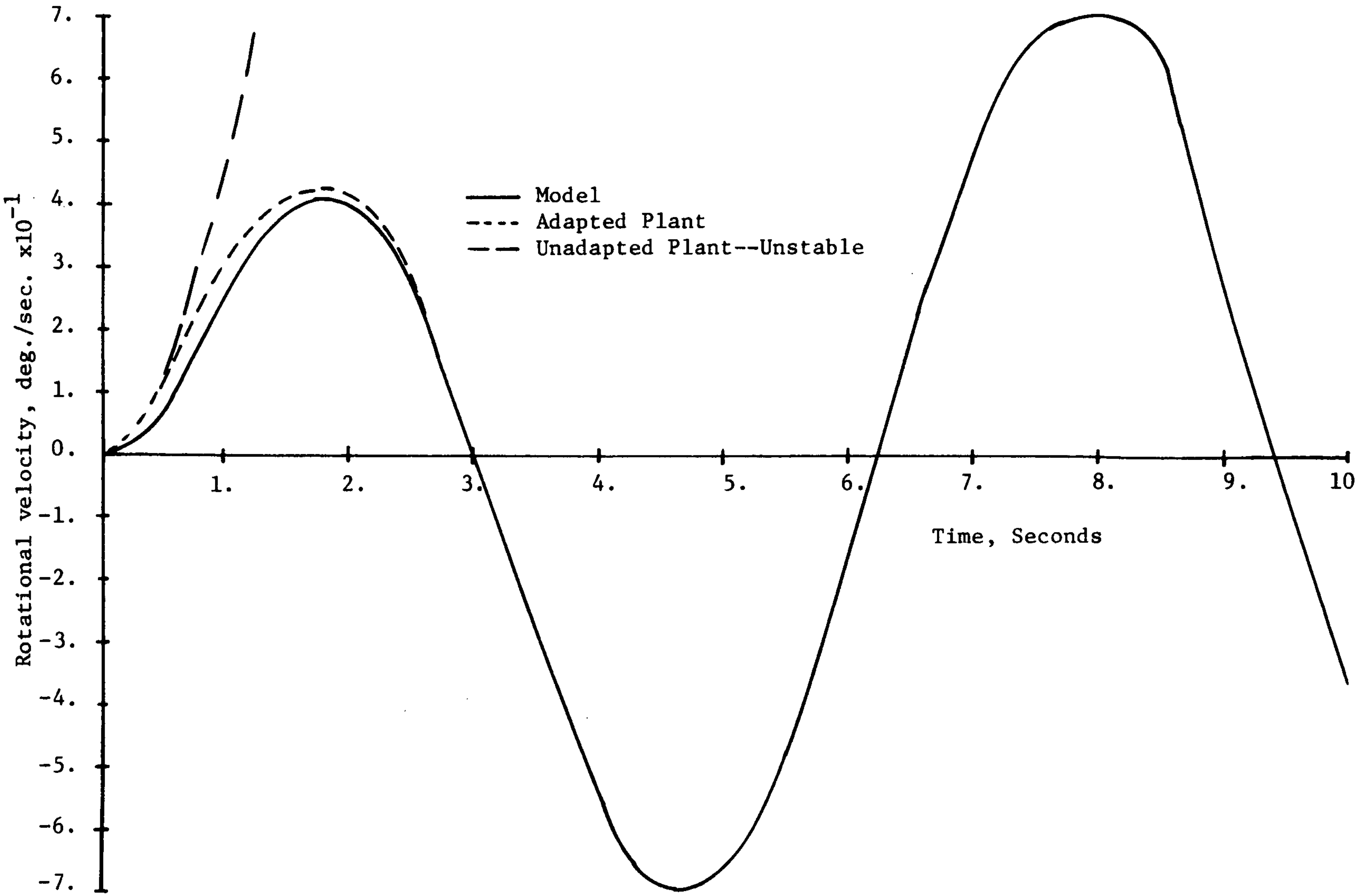
IV-15. Bending mode responses of model, adapted plant, and unadapted plant, positive wind gust, frequency-domain technique.



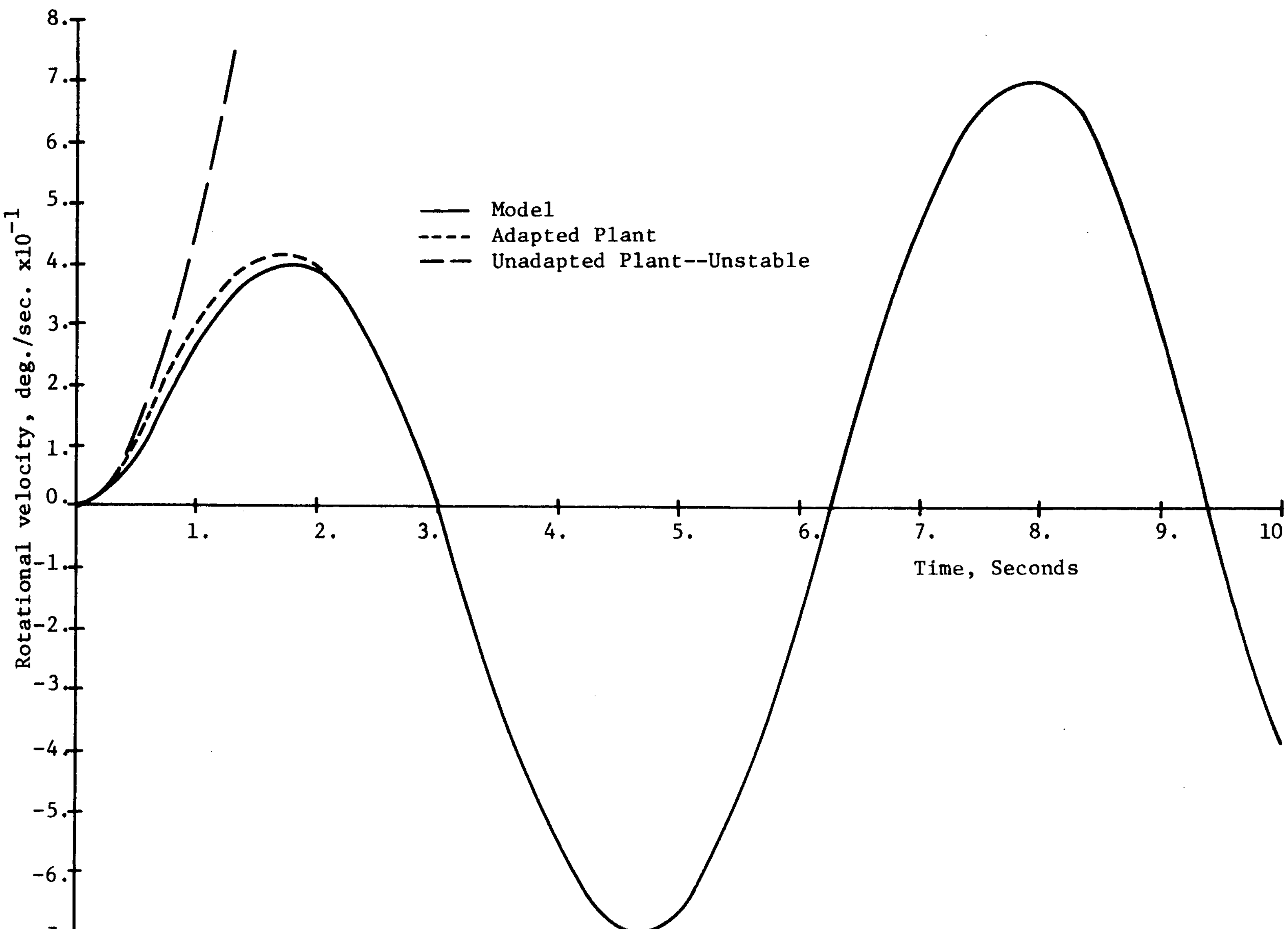
IV-16. Rotational position responses of model, adapted plant, and unadapted plant, positive wind gust, frequency-domain technique.



IV-17. Rotational position responses of model, adapted plant, and unadapted plant, negative wind gust, frequency-domain technique.



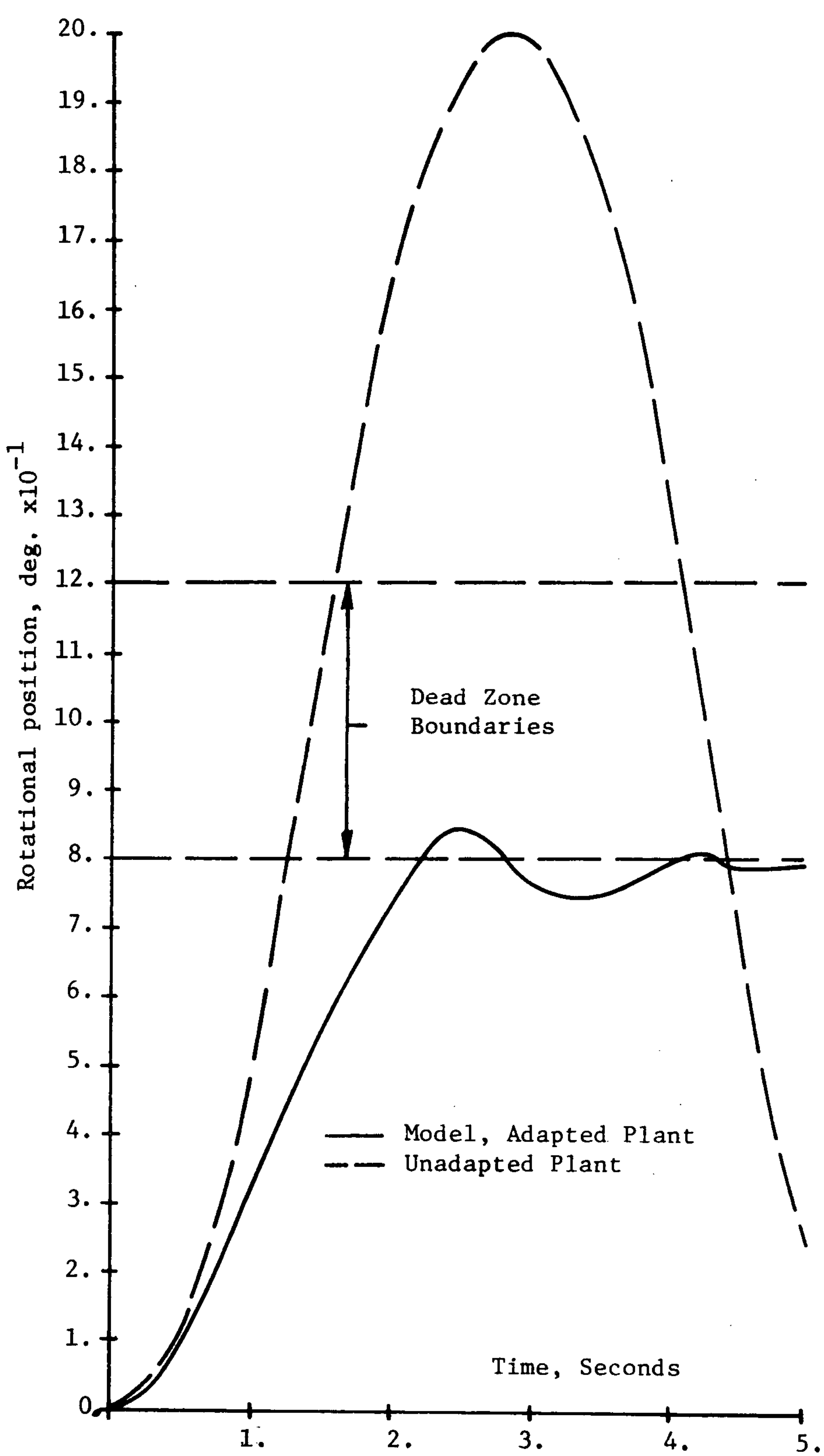
IV-18. Rotational velocity responses of model, adapted plant, and unadapted plant, positive wind gust, frequency-domain technique.



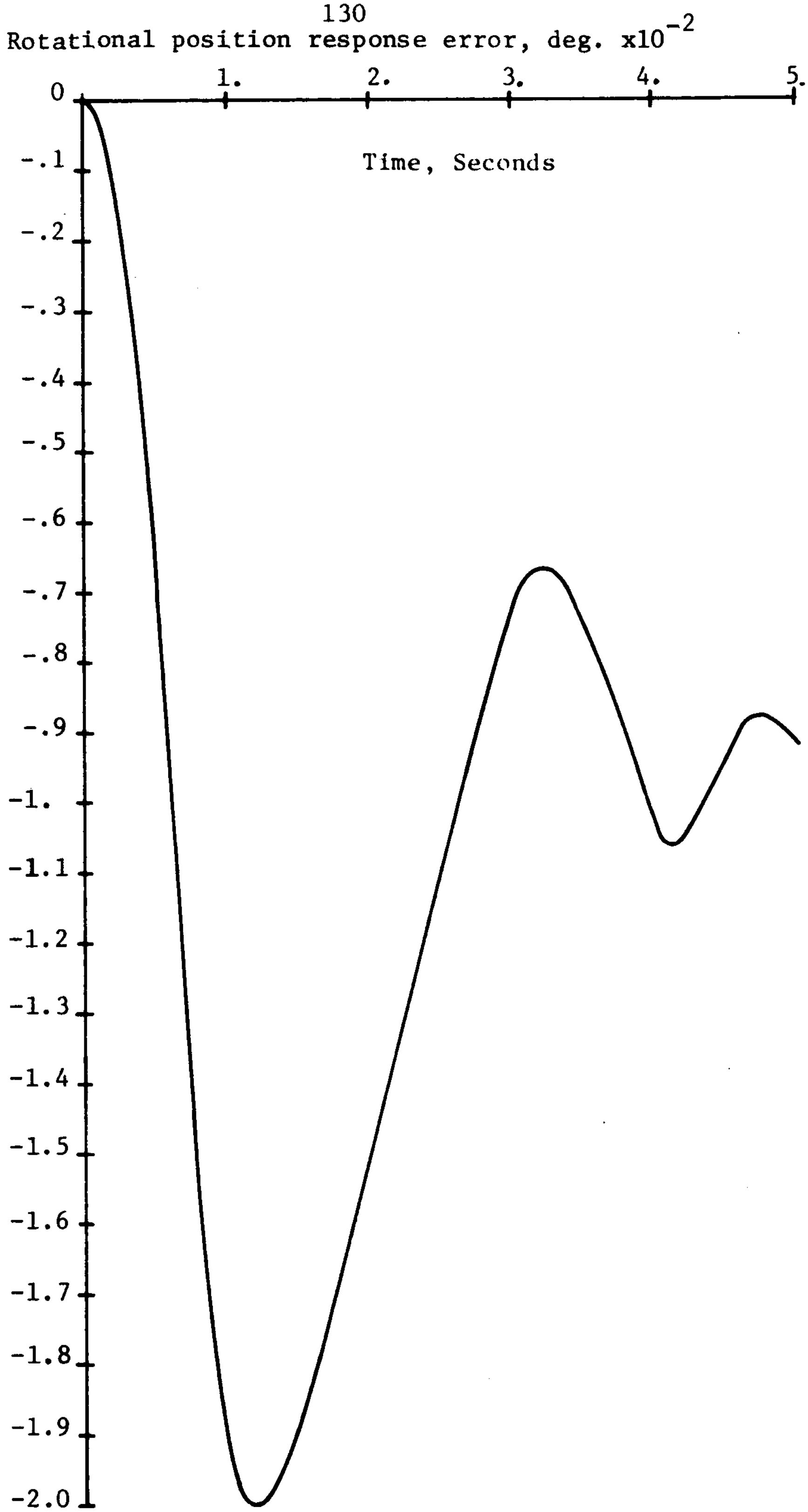
IV-19. Rotational velocity responses of model, adapted plant, and unadapted plant, negative wind gust, frequency-domain technique.

Thus, even for large wind gust disturbances, the system adapts so that the plant tracks the model very accurately.

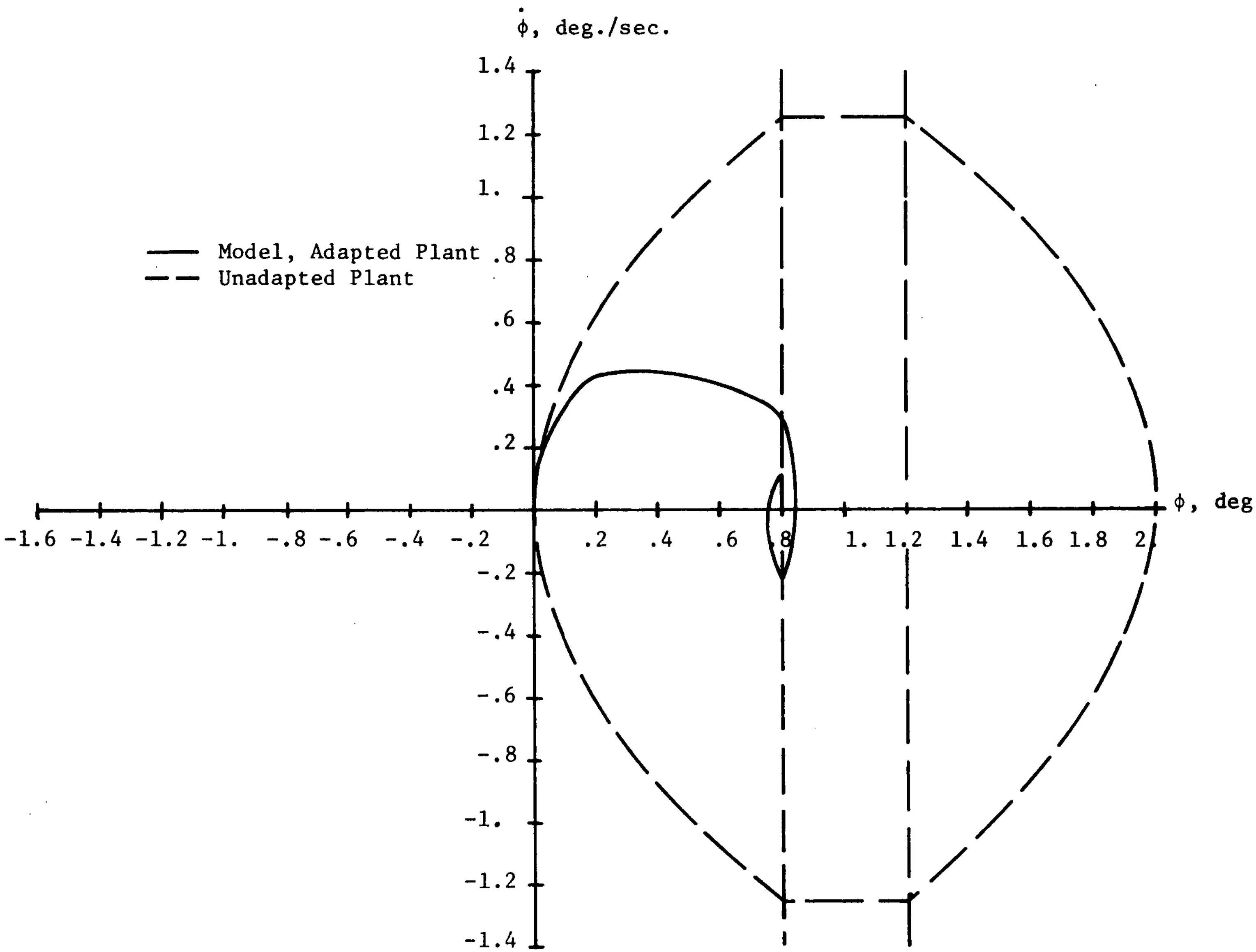
The frequency-domain design technique was investigated, as was the time-domain technique of Chapter III, for the Space Shuttle vehicle with the thrust characteristics represented by a contactor mechanism with a dead-zone. Figure IV-20 shows the rotational position response versus time for the unadapted plant, the adapted plant, and the model with a step input command to the system. As can be seen from the figure, the unadapted plant response is substantially different from the adapted response whereas the adapted response and the model response are virtually indistinguishable. The actual rotational position response error is shown in Figure IV-21 and is seen to be quite small. Figure IV-22 shows a phase-plane plot for the unadapted plant and the adapted plant. As was shown in Chapter III, the unadapted response is oscillatory. However, the adapted plant responds in a very satisfactory manner as the plant closely tracks the model. Figure IV-23 shows the rotational position response for the unadapted plant, the adapted plant, and the model with an initial condition on the plant. As with the previous case, the adapted plant and model responses are virtually the same whereas the unadapted plant response is grossly different. Figure IV-24 shows the actual rotational position error as a function of time. This shows that the response error is very small which implies that the plant is tracking the model very accurately. Figure IV-25 shows a phase-plane



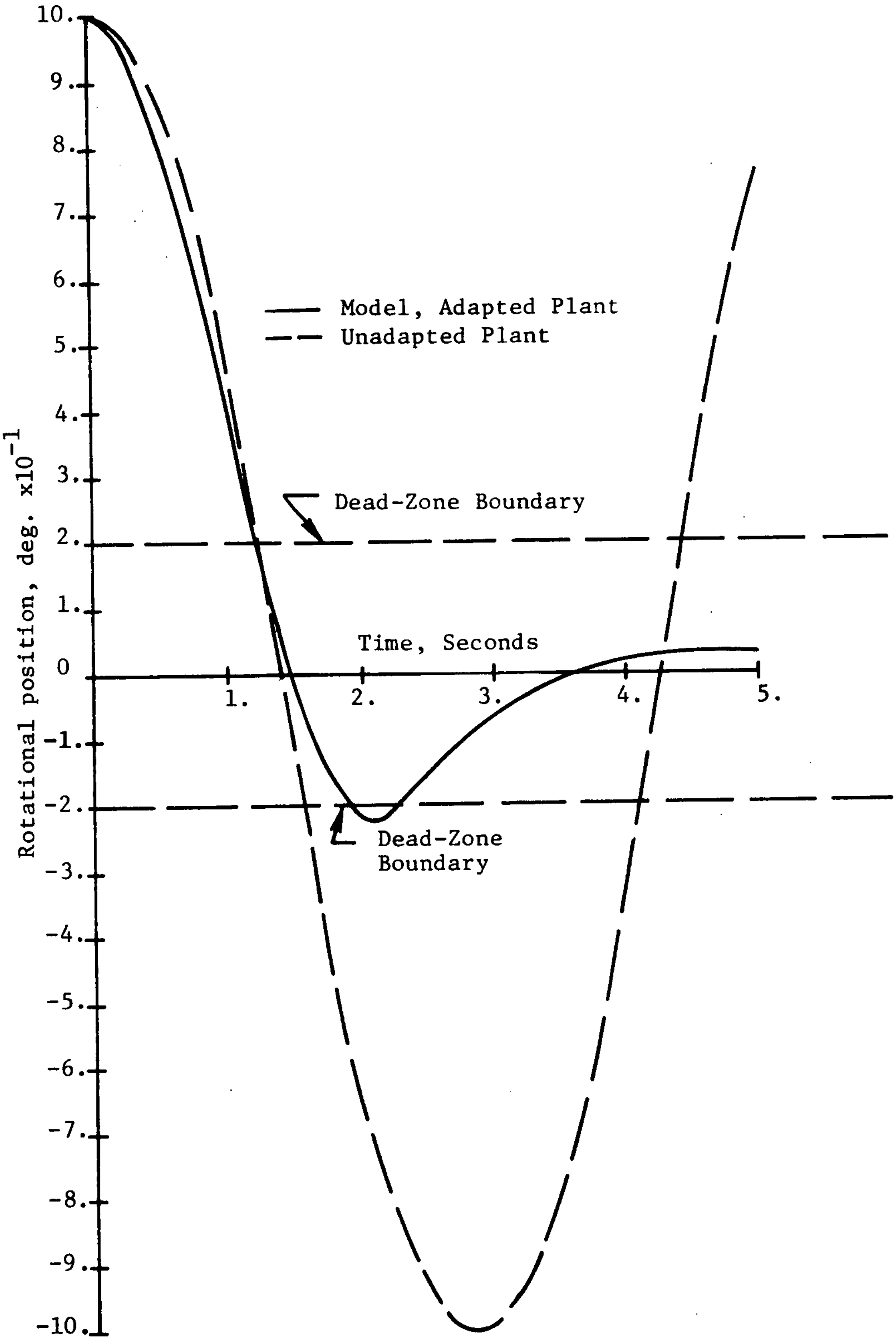
IV-20. Rotational position response curves of model, adapted plant, and unadapted plant, step command, frequency-domain technique.



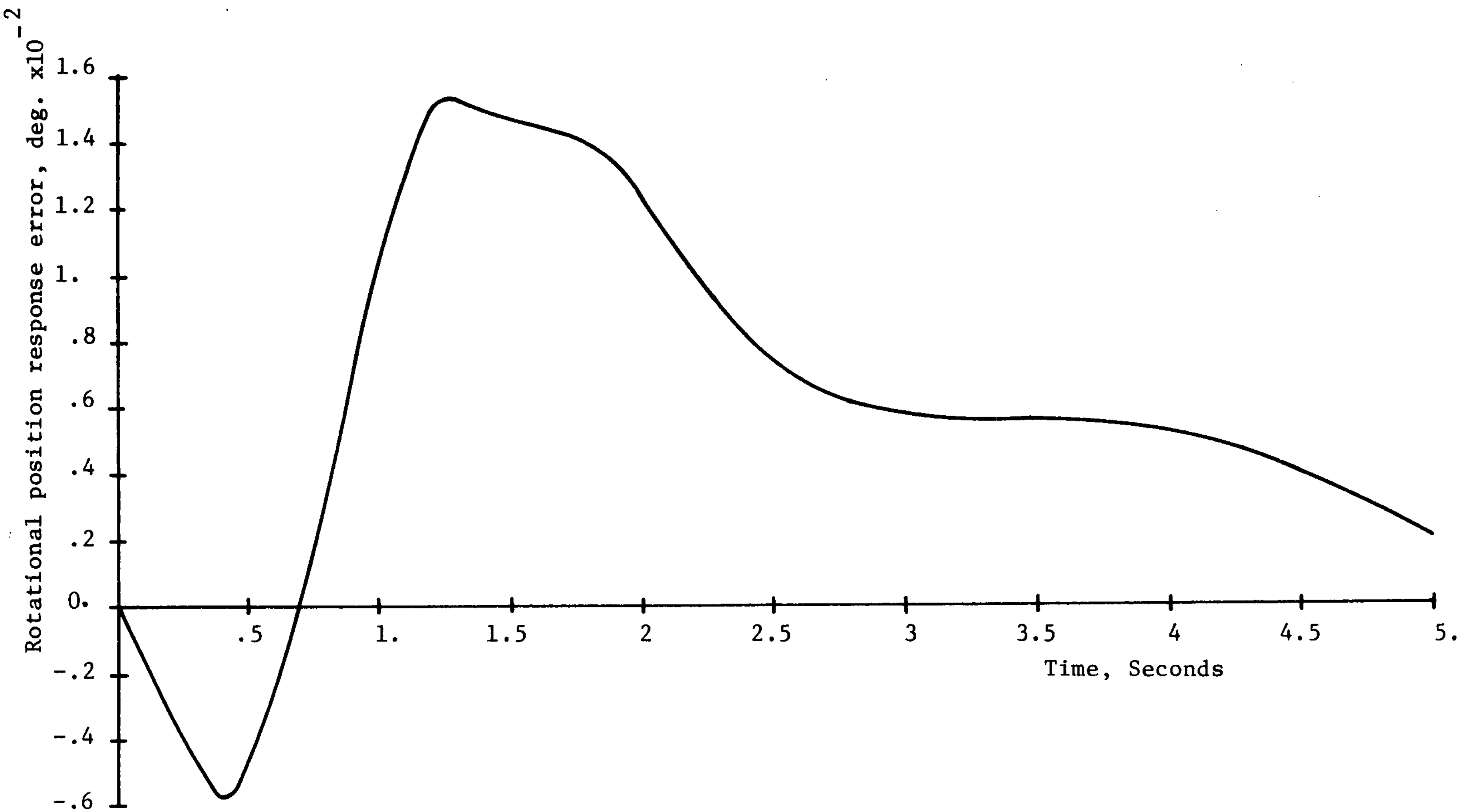
IV-21. Rotational position response error versus time, step command, frequency-domain technique.



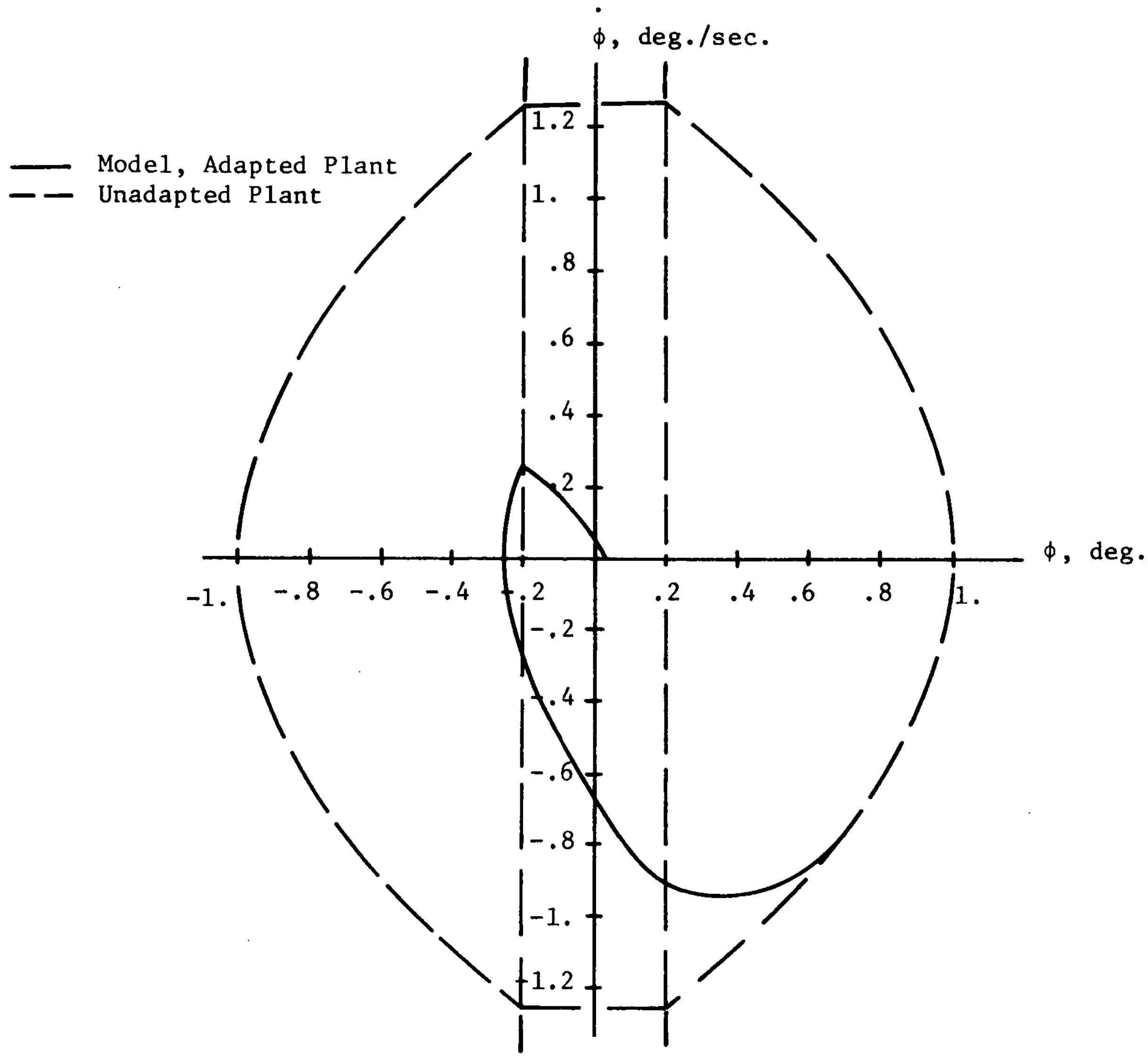
IV-22. Phase-plane plot of model, adapted plant, and unadapted plant, step command, frequency-domain technique.



IV-23. Rotational position response curves of model, adapted plant, and unadapted plant, initial condition, frequency-domain technique.



IV-24. Rotational position response error versus time, initial condition, frequency-domain technique.



IV-25. Phase-plane plot of model, adapted plant, and unadapted plant, initial condition, frequency-domain technique.

plot for the adapted plant and the unadapted plant. As was shown previously, the unadapted system is highly oscillatory. However, the adapted system, using the frequency-domain design technique, is well damped and tracks the model with little error.

V. AN APPLICATION OF SENSITIVITY COEFFICIENTS TO THE DESIGN OF MODEL-REFERENCE ADAPTIVE CONTROL SYSTEMS

The purpose of this chapter is to discuss an application of sensitivity coefficients [16], [17], and [18]. Sensitivity coefficients are shown to be applicable to the design of model-reference adaptive control systems. In particular, they are applied to the adaption of the Space Shuttle vehicle which was investigated in Chapters III and IV.

In this application of sensitivity coefficients, it is assumed that the plant parameters while being unknown, are constant. The model response, as in all model-reference adaptive control schemes, is considered to be the desired response. The object of the adaption process is to use the sensitivity coefficients in a manner so as to change some of the plant parameters, which in general will have non-ideal values, so as to cause the plant to track the model.

A. Definition of Sensitivity Coefficient

Let us now define the sensitivity coefficient as in [19]. Let the output of a system be denoted by $x(t, p_1, p_2)$ where p_1 and p_2 are nominal parameter values. If the parameter p_1 varies from the nominal value by some value Δp_1, the output is then given by $x(t, p_1 + \Delta p_1, p_2)$. The sensitivity coefficient is defined by:

$$\lim_{\Delta p_1 \to 0} \frac{x(t, p_1 + \Delta p_1, p_2) - x(t, p_1, p_2)}{\Delta p_1} \qquad \text{(V-1)}$$

which is the partial derivative of $x(t, p_1, p_2)$ with respect to p_1 evaluated at the nominal parameter values. Thus, the sensitivity coefficient for p_k is the partial derivative of the system output with respect to a system parameter p_k and will be denoted by $\frac{\partial x}{\partial p_k}$.

B. Use of Sensitivity Coefficients in Model-Reference Adaptive Control System Design

One model-reference adaptive control method in which sensitivity coefficients may be utilized is a technique developed by Osburn [8]. Osburn defines the optimum performance as that which results when the system parameters are adjusted to produce a minimum of some specified Performance Index. The performance index selected was:

$$\text{P. I.} = \int_{t_o}^{t_1} f(e)dt \tag{V-2}$$

The interval $[t_o, t_1]$ is of sufficient length to include most of the dynamic response of the system for an input at time t_o and $f(e)$ is an even function of the error. The function selected for investigation by Osburn was

$$f(e) = 1/2\ e^2 \tag{V-3}$$

It was then concluded that each adjustable parameter, p_k, should be changed continuously at a rate proportional to the negative of the partial derivative of $f(e)$ with respect to p_k. In equation form, this is given by:

$$\dot{p}_k = -S_k f'(e) \frac{\partial e}{\partial p_k} \tag{V-4}$$

where S_k is a constant. For the f(e) used, which is given in Equation (V-3), the adaptive parameter rate may be written as:

$$\dot{p}_k = -S_k e \frac{\partial e}{\partial p_k} \tag{V-5}$$

Now consider the term $\frac{\partial e}{\partial p_k}$. For the model-reference adaptive control system, the response error is given by:

$$e = x_m - x_p \tag{V-6}$$

Taking the partial derivative of Equation (V-6) with respect to the parameter p_k gives

$$\frac{\partial e}{\partial p_k} = -\frac{\partial x_p}{\partial p_k} \tag{V-7}$$

The term $\frac{\partial x_m}{\partial p_k}$ is equal to zero because the model does not contain any adaptive parameters. Examination of Equation (V-7) shows that the term $\frac{\partial e}{\partial p_k}$ is the negative of the previously defined sensitivity coefficient. Thus, the rate for the adaptive parameter p_k may be rewritten in terms of the response error and the sensitivity coefficient as:

$$\dot{p}_k = S_k e \frac{\partial x_p}{\partial p_k} \tag{V-8}$$

Integrating Equation (V-8), the adaptive parameters are:

$$p_k = S_k \int_{t_0}^{t_1} (e.\frac{\partial x_p}{\partial p_k}) d\pi + p_k(t_o) \tag{V-9}$$

In general, however, it is not possible to determine $\frac{\partial x_p}{\partial p_k}$ because this requires a knowledge of the plant parameter values and this lack of information is one of the primary motivations for using an adaptive system. Thus, it is necessary to approach the problem in a different manner. Hence, the following assumption is made [11]: If the response of the plant approximates that of the model, then the sensitivities of the model approximate the true sensitivities of the plant. With this assumption, the model parameter values may then be used in order to evaluate the sensitivity coefficients necessary for the implementation of Equation (V-9).

Although Equation (V-9) provides a method for system adaptation, it should be noted that there is no provision for insuring the stability of the response error as was done in the design techniques of Chapters III and IV.

C. Generation of Sensitivity Coefficients

The sensitivity coefficients necessary for implementation of the adaptive scheme will be generated as in [19]. Consider the general case of a system described in the frequency domain by

$$\frac{C(s)}{R(s)} = \frac{N}{D} = \frac{s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{s^m + a_{m-1}s^{m-1} + \ldots + a_1 s + a_0} \qquad \text{(V-10)}$$

The partial derivative of C with respect to a parameter p_j is

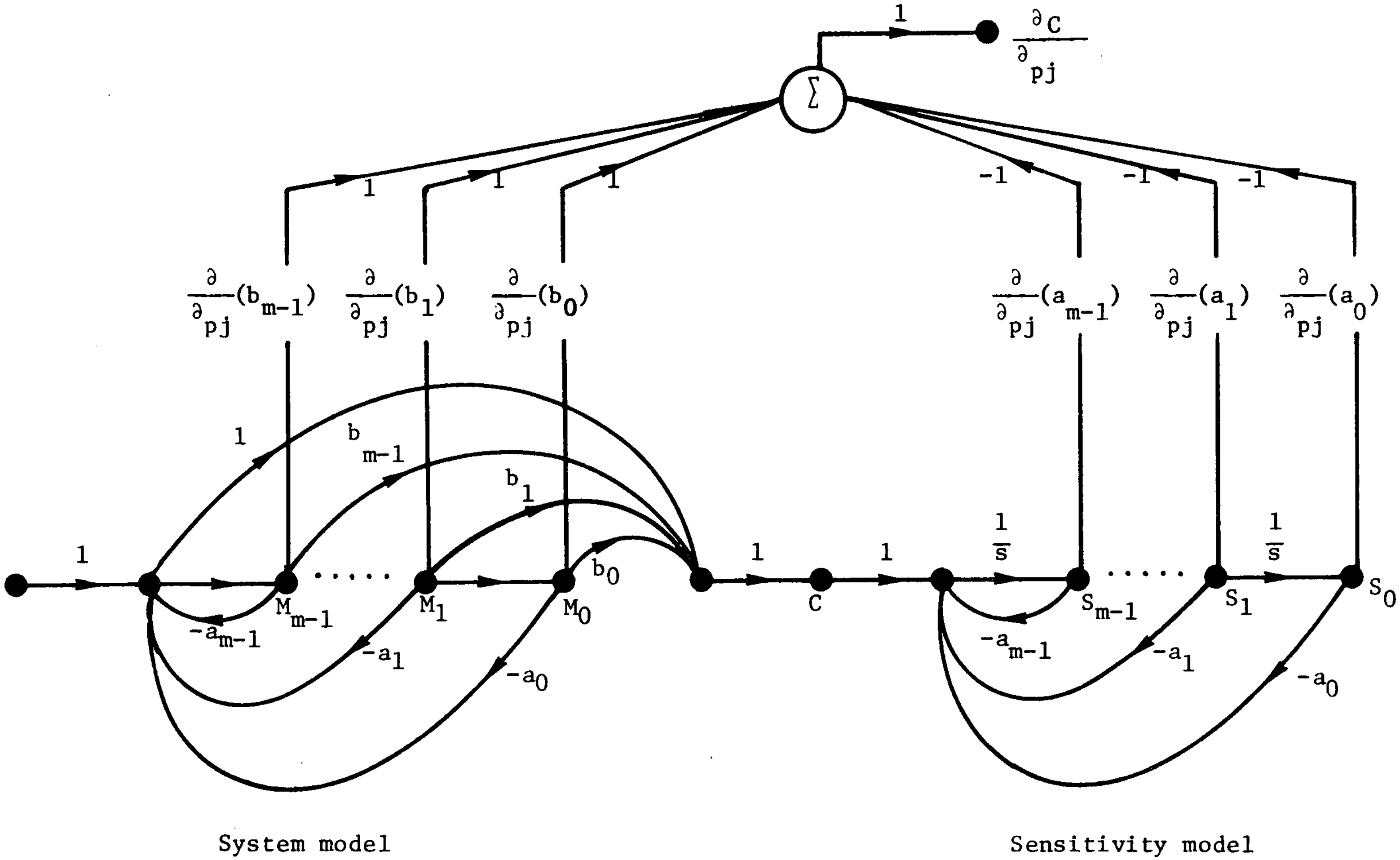
$$\frac{\partial C}{\partial p_j} = \frac{R(D \frac{\partial N}{\partial p_j} - N\frac{\partial D}{\partial p_j})}{D^2} \qquad \text{(V-11)}$$

By using Equation (V-10), Equation (V-11) may be rewritten as:

$$\frac{\partial C}{\partial p_j} = \frac{R}{D}\frac{\partial n}{\partial p_j} - \frac{C}{D}\frac{\partial D}{\partial p_j} \qquad \text{(V-12)}$$

A signal-flow graph of Equation (V-12) is shown in Figure V-1. Thus, if the input to the system in Figure V-1 is R, then the sensitivity coef-fient $\frac{\partial C}{\partial p_j}$ is a summation of the signals present at the M_ℓ, $\ell = 0, \ldots, m-1$ nodes of the system model and the signals present at the S_ℓ, $\ell = 0, \ldots,$ m-1 nodes of the sensitivity model in accordance with the equation:

$$\frac{\partial C}{\partial p_j} = \sum_{\ell=0}^{m-1} (M_\ell \frac{\partial b_\ell}{\partial p_j} - S_\ell \frac{\partial a_\ell}{\partial p_j}) \qquad \text{(V-13)}$$



V-1. Signal-flow graph showing system model and sensitivity model.

The $\frac{\partial b_\ell}{\partial p_j}$ and $\frac{\partial a_\ell}{\partial p_j}$ are evaluated at the nominal values of the parameter set.

D. Application to Space Shuttle Vehicle

The design procedure defined by Equation (V-8) was applied to the Space Shuttle vehicle. The model and plant are as described in Section D of Chapter IV and which are repeated here for convenience. The bending mode plant and model, $G_{Ep}(s)$ and $G_{Em}(s)$, respectively, are:

$$\frac{X_{Ep}(s)}{R(s)} = G_{Ep}(s) = \frac{s}{s^2 + (.2 + p_{2E})s + 4} \tag{V-14}$$

and

$$\frac{X_{Em}(s)}{R(s)} = G_{Em}(s) = \frac{s}{s^2 + 1.414s + 1} \tag{V-15}$$

For the rotational mode, the plant and model transfer functions, denoted by $G_{pp}(s)$ and $G_{pm}(s)$, respectively, are:

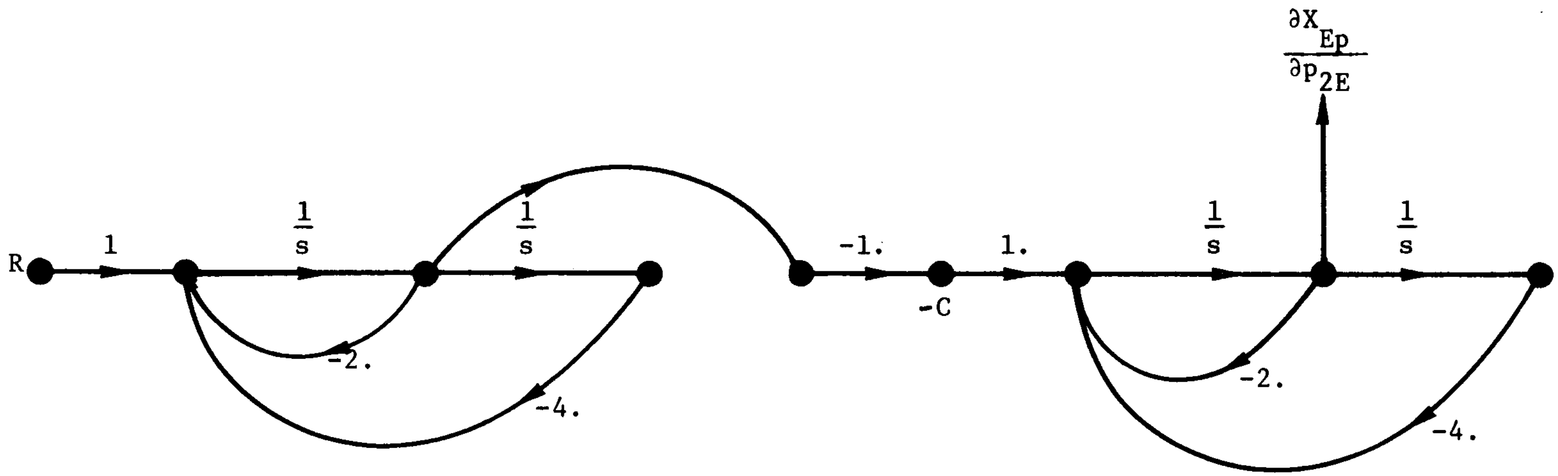
$$\frac{X_{pm}(s)}{R(s)} = G_{pp}(s) = \frac{s + 1}{s^2 + p_{2p}s + p_{1p}} \tag{V-16}$$

and

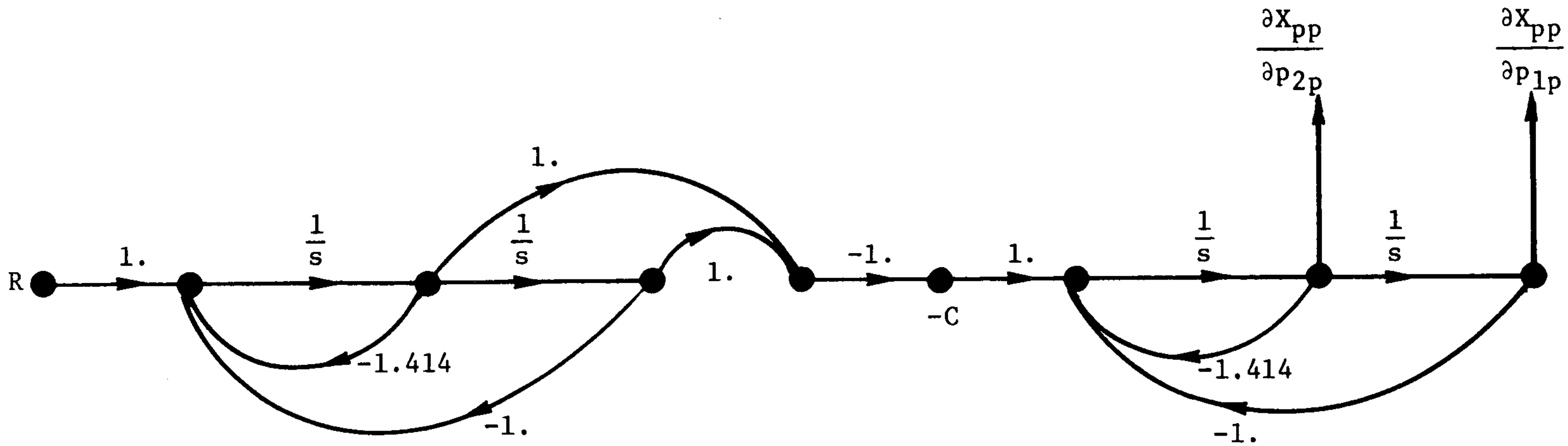
$$\frac{X_{pm}(s)}{R(s)} = G_{pm}(s) = \frac{s + 1}{s^2 + 1.414s + 1} \qquad \text{(V-17)}$$

Applying the methods of Section C, the sensitivity coefficient $\frac{\partial X_{Ep}}{\partial p_{2E}}$ may be generated as shown in Figure V-2 and the sensitivity coefficients $\frac{\partial X_{pp}}{\partial p_{1p}}$ and $\frac{\partial X_{pp}}{\partial p_{2p}}$ may be generated as shown in Figure V-3.

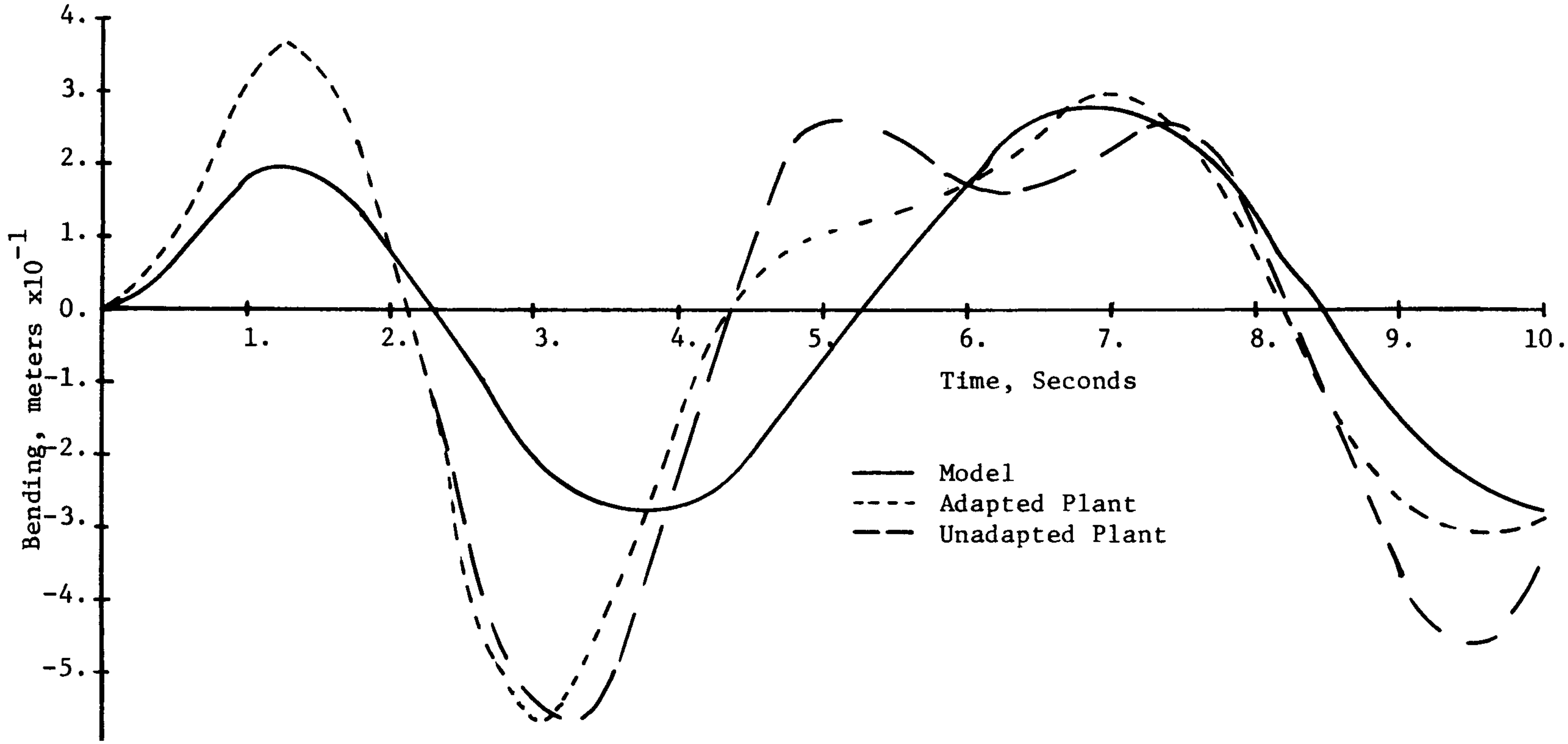
The method utilizing sensitivity coefficients was applied to the Space Shuttle vehicle assuming first a sine wave input and then a step input. Figures V-4 and V-5 show the bending mode responses for the model, the adapted plant, and the unadapted plant for a sine wave input and a step input, respectively. As can be seen from the figures, adaption is occuring, but at a slow convergence rate. Figures V-6 and V-7 show the rotational position and velocity responses for the model, the adapted plant, and the unadapted plant for a sine wave input. The figures show that adaption is occuring, but at a slow rate. Several seconds are necessary before the adaption has a noticeable effect on the system. Figures V-8 and V-9 show the rotational position and velocity responses for the model, the adapted plant, and the unadapted plant for a step input to the system. Again, the figures show that the adaption convergence is slow. The adapted system is much more oscillatory than the model, but the plant does track the model after a



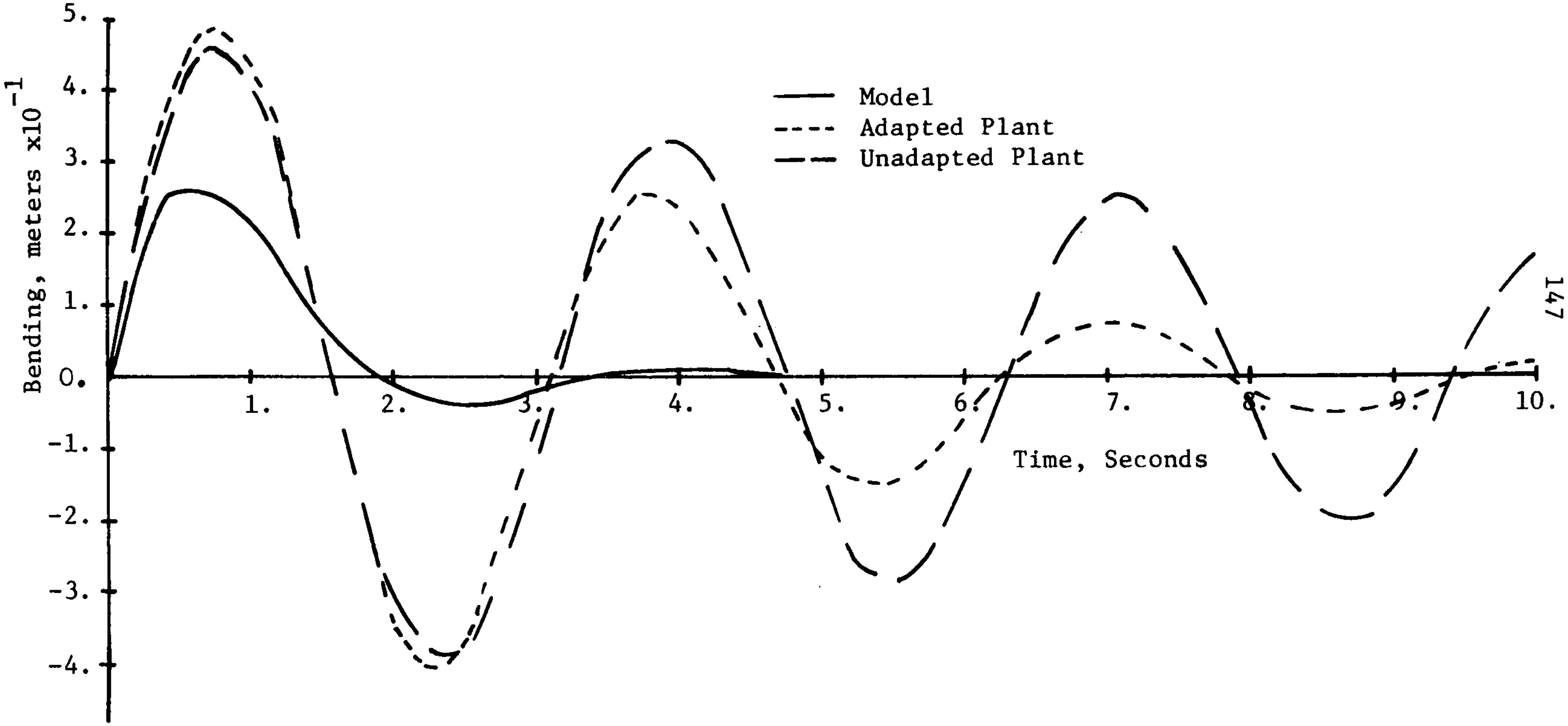
V-2. Signal-flow graph for generation of $\frac{\partial X_{Ep}}{\partial p_{2E}}$.



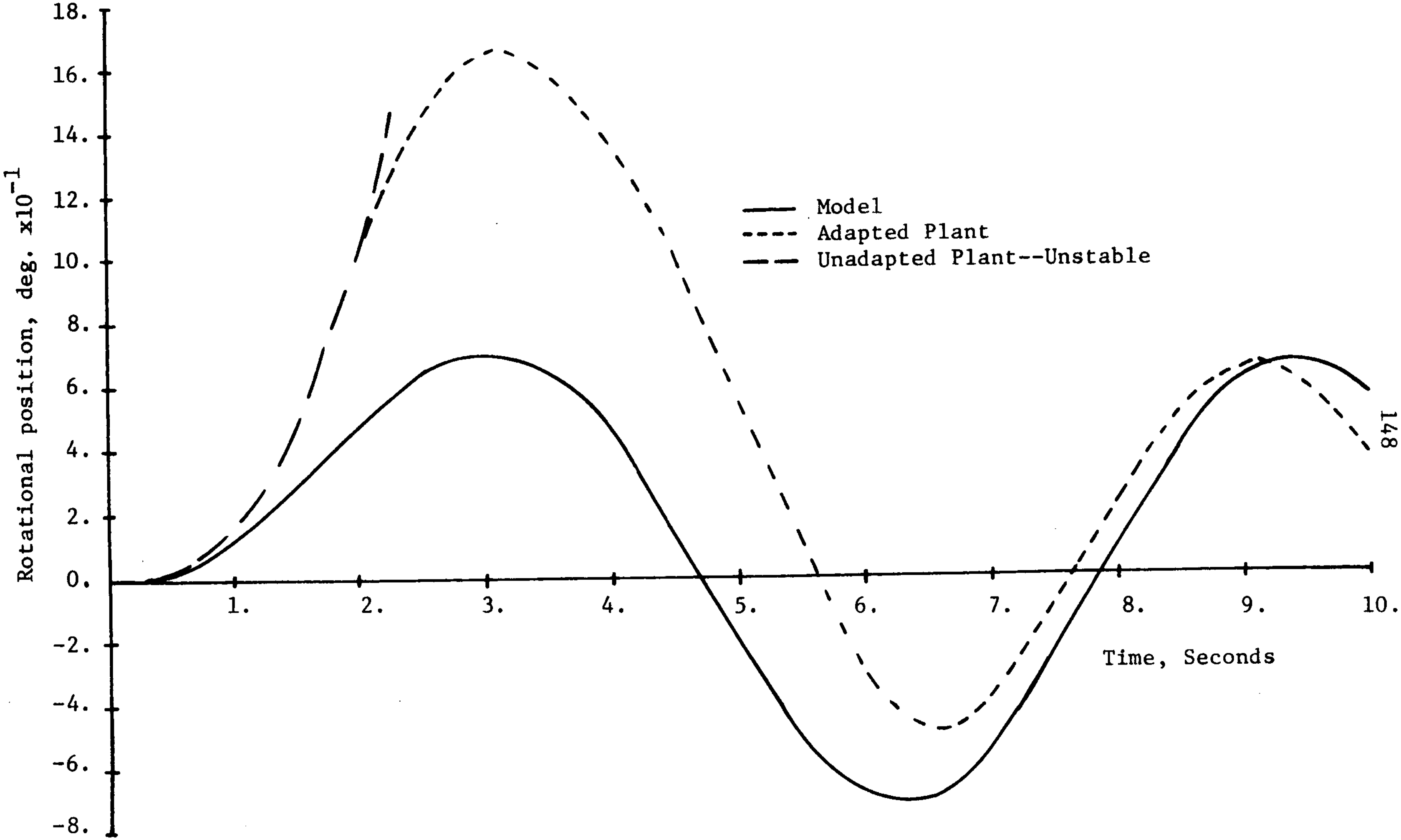
V-3. Signal-flow graph for generation of $\frac{\partial X_{pp}}{\partial p_{1p}}$ and $\frac{\partial X_{pp}}{\partial p_{2p}}$.



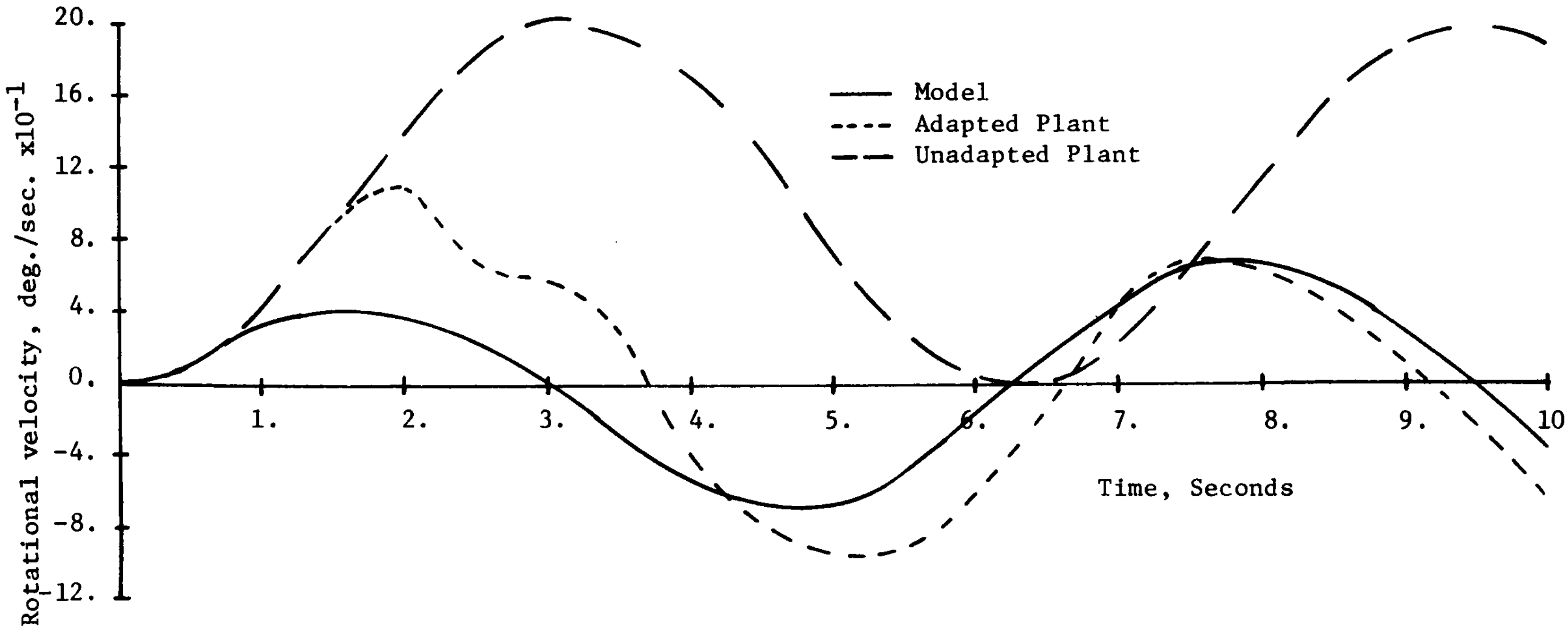
V-4. Bending mode responses of model, adapted plant, and unadapted plant, sine wave input, sensitivity coefficient method.



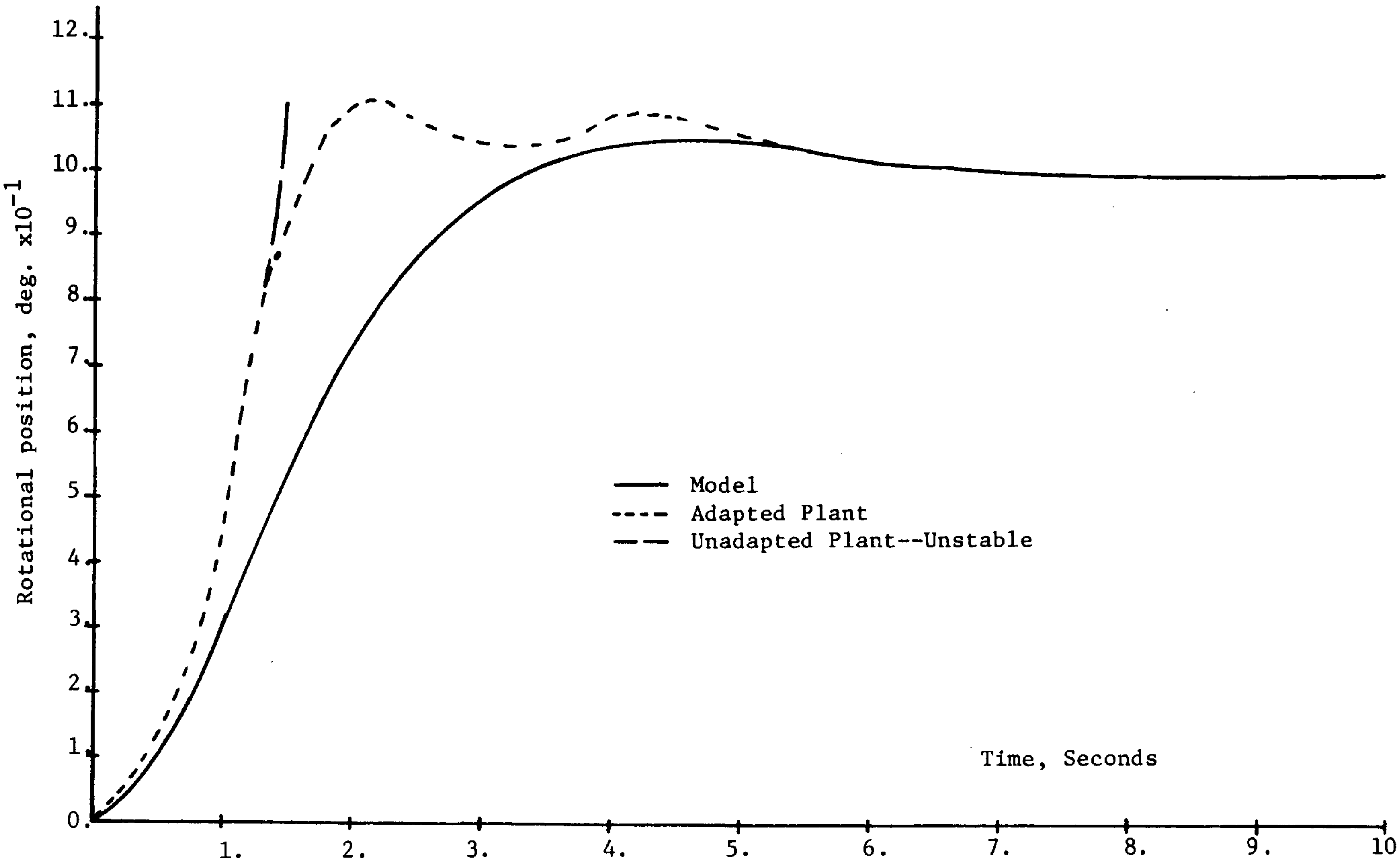
V-5. Bending mode responses of model, adapted plant, and unadapted plant, step input, sensitivity coefficient method.



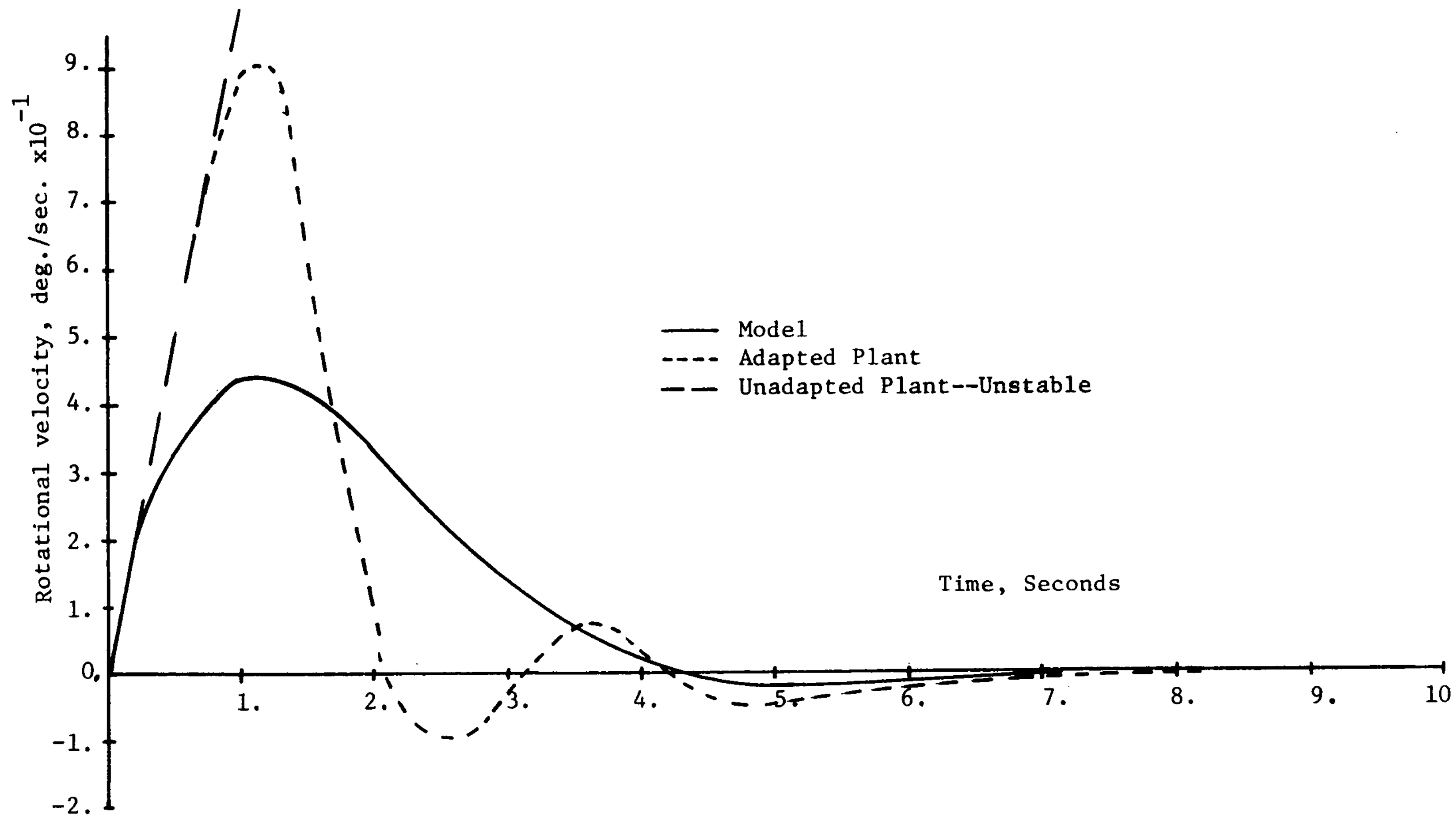
V-6. Rotational position responses of model, adapted plant, and unadapted plant, sine wave input, sensitivity coefficient method.



V-7, Rotational velocity responses of model, adapted plant, and unadapted plant, sine wave input, sensitivity coefficient method.

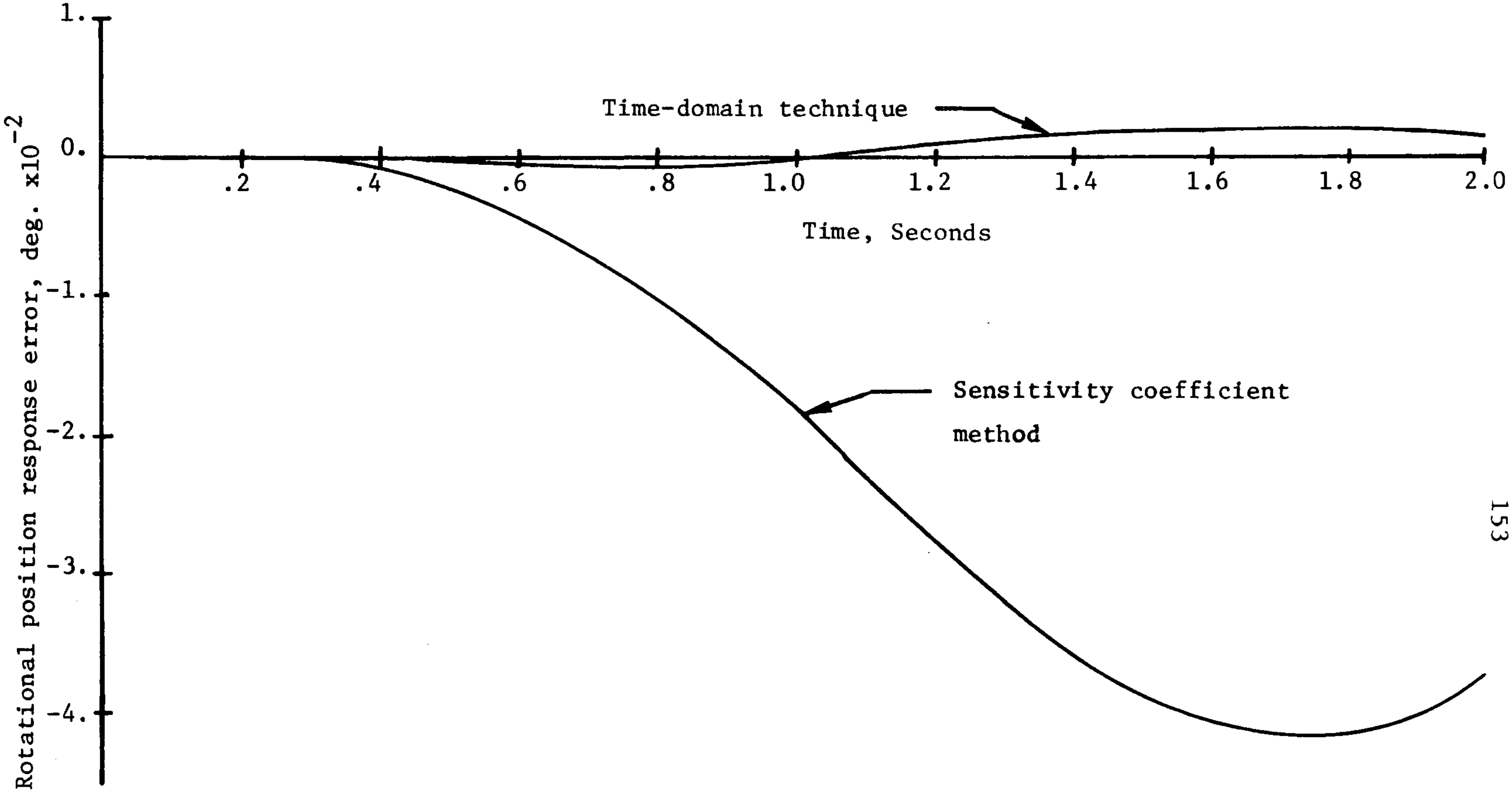


V-8. Rotational position responses of model, adapted plant, and unadapted plant, step input, sensitivity coefficient method.



V-9. Rotational velocity responses of model, adapted plant, and unadapted plant, step input, sensitivity coefficient method.

sufficient period of time. Figure V-10 shows plots of the rotational position response error versus time for the Space Shuttle vehicle adapted with the method utilizing sensitivity coefficients and with the time-domain technique developed in Chapter III. Clearly, the response error is significantly less with the time-domain technique.



V-10. Rotational position response error for time-domain adaptation and sensitivity coefficient adaptation.

VI. CONCLUSIONS

In this report, the problems associated with the design of model-reference adaptive control systems were considered and solutions to these problems were advanced. Stability of the adapted system was a primary consideration in the design techniques developed. Thus, Liapunov's direct method formed an integral part of the development of both the time-domain and frequency-domain design techniques.

The basic stability definitions, theorems regarding Liapunov stability and their application as well as a justification for the use of Liapunov's direct method in the design of model-reference adaptive control systems are presented in Chapter II.

A time-domain model-reference adaptive control design technique is presented in Chapter III. This represents a significant extension of previous design techniques. With the design technique developed in Chapter III, adaption to the system is a function of the response error and the plant states as well as the derivative and integral of these quantities. Results obtained from the application of this design technique to the Space Shuttle vehicle show that the response error is less, the plant more favorably tracks the model and, in general, better system adaptation is achieved than with previous methods. Results also show that very successful adaptation is obtained even with large wind gusts. Good adaptation is also achieved with non-linear system characteristics as well as with linear system characteristics.

A frequency-domain model-reference design technique is presented in Chapter IV. Physically realizable feedback filters, pre-filters and the filter gains are designed so as to provide system adaptation. The approach to the problem was influenced by the fact that an adaptive system in which the same basic filter networks could be utilized for a number of missions was desired. As for the time-domain design method of Chapter III, stability of the adapted system was incorporated in the development of the design technique via Liapunov's method. Results obtained from the application of this design technique to the Space Shuttle vehicle showed that excellent system adaptation was obtained and the plant, even with severe wind gusts, successfully tracks the model. Non-linear as well as linear system characteristics were studied and the results showed that system adaptation was excellent for either case.

Sensitivity coefficients were shown, in Chapter V, to be applicable to the design of model-reference adaptive control systems. Results obtained from the application of a design technique utilizing sensitivity coefficients to the Space Shuttle vehicle with linear characteristics showed that system adaptation was achieved. However, the adaptation is much slower and causes a much larger response error than either of the design techniques developed in Chapters III and IV.

The use of multiple force points should be investigated. For example, in addition to the on-off thrust characteristics of a reaction jet control system, the use of control forces available from aerodynamic surfaces may be desirable. The inter-relationship of these various forces on the adapted system should be investigated. The investigation of optimal adaptive system convergence rates and their relationship with the dead-zone boundaries of the adapted system should also be investigated. Another item of much practical significance would be a study to determine the range of variations of various system parameters such as mass, moment of inertia, center of gravity, etc., that could be tolerated by the adaptive system without having to do additional detailed pre-flight studies. A study to determine the technique that could be most easily implemented on a digital computer is also recommended.

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